Ensemble Transform Kalman Filter
- L02. Inflations and Implementations -

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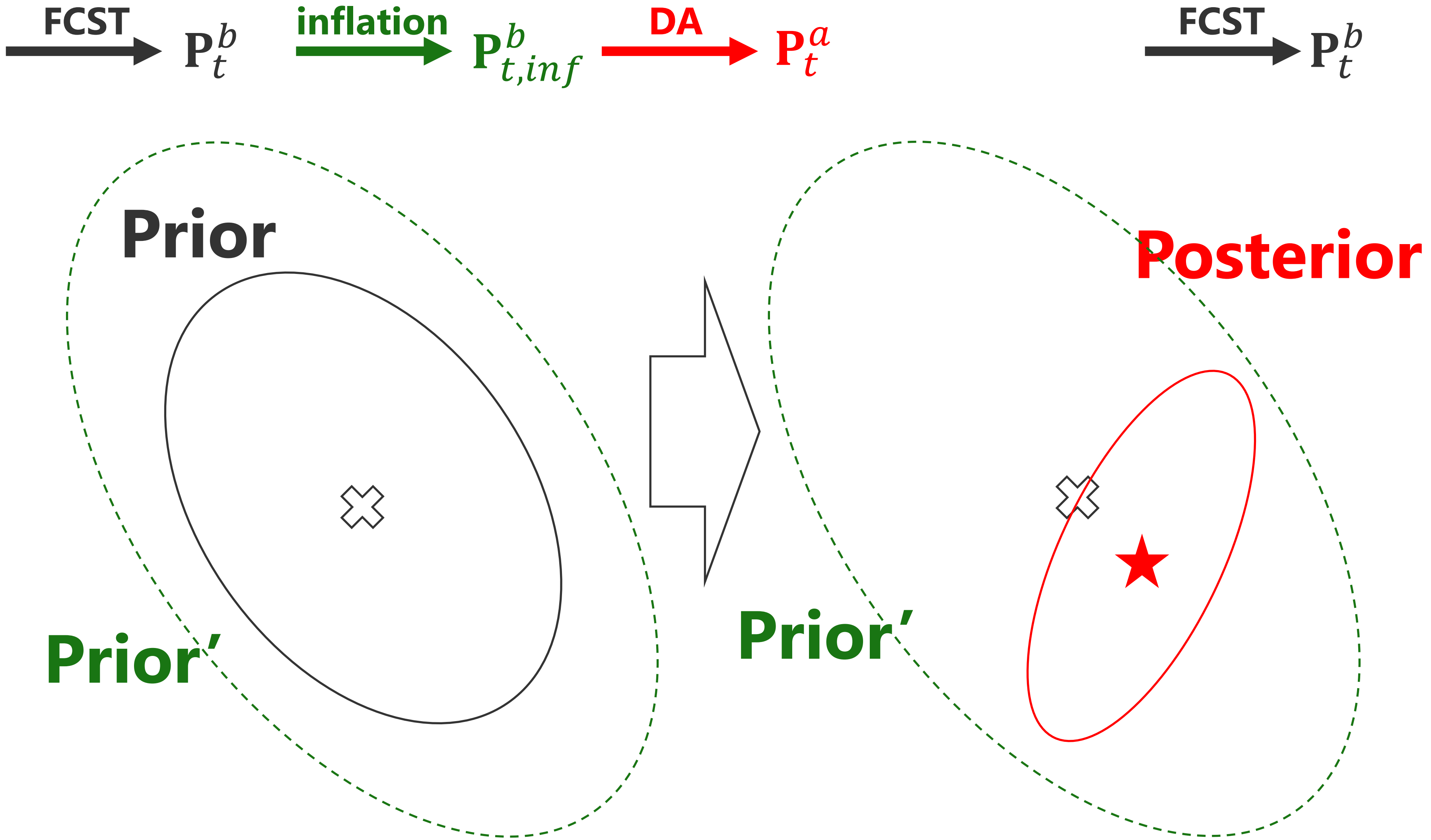
Multiplicative Inflation

Anderson and Anderson (1999; MWR)

Multiplicative Inflation $\mathbf{P}^b_{t,inf} = \Delta \mathbf{P}^b_t$



RTPP

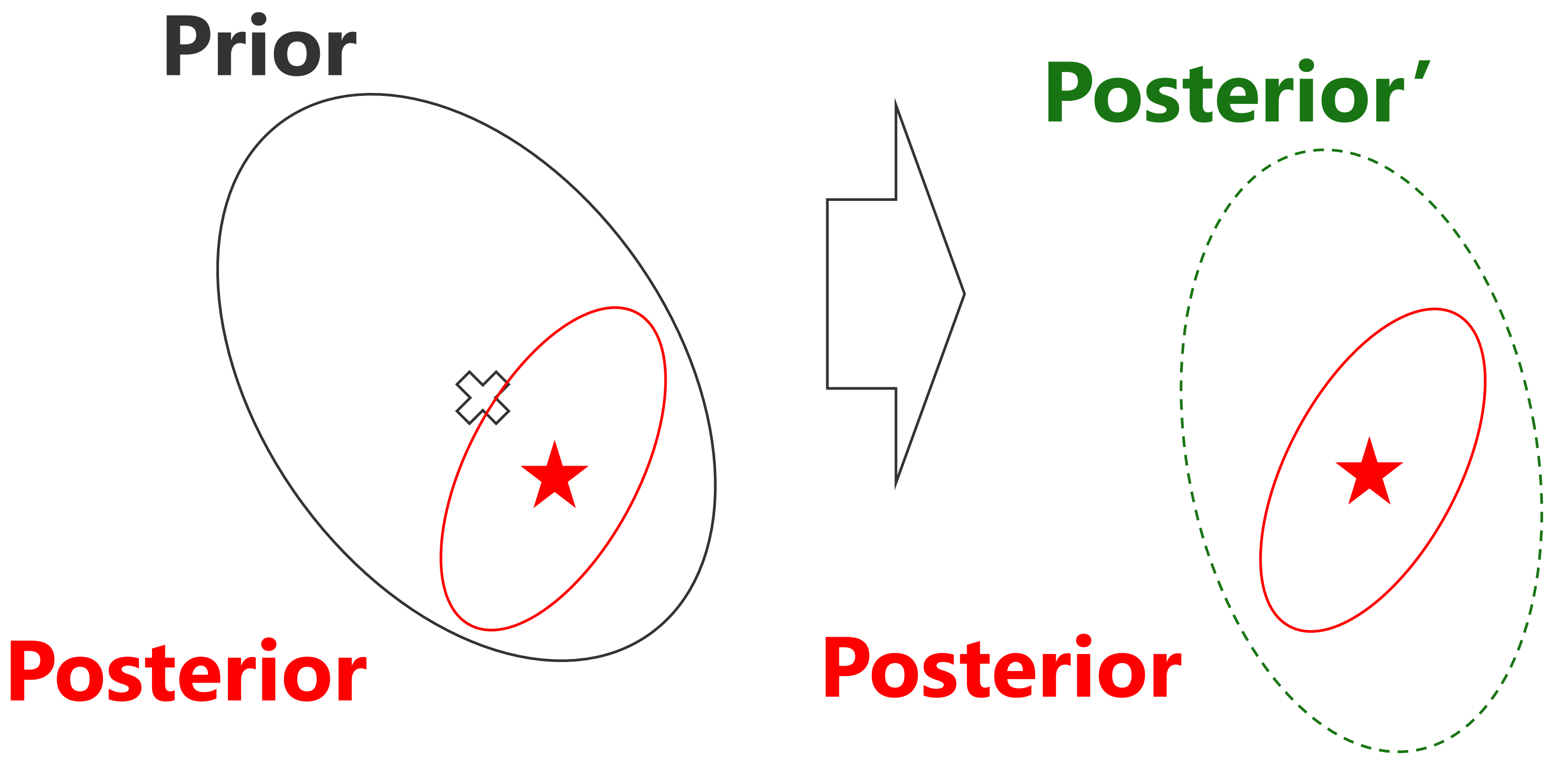
Zhang et al. (2004; MWR)

Relaxation to Prior Perturbation (RTPP)

$$\delta\mathbf{X}_{t,inf}^{a} = (1-\alpha)\delta\mathbf{X}_{t}^{a} + \alpha\delta\mathbf{X}_{t}^{b}$$

$\xrightarrow{\text{FCST}} \mathbf{P}_t^b$

$\xrightarrow{\text{DA}} \mathbf{P}_t^a \xrightarrow{\text{relaxation}} \mathbf{P}_{t,inf}^a \xrightarrow{\text{FCST}} \mathbf{P}_t^b$



RTPS

Whitaker and Hamill (2012; MWR)

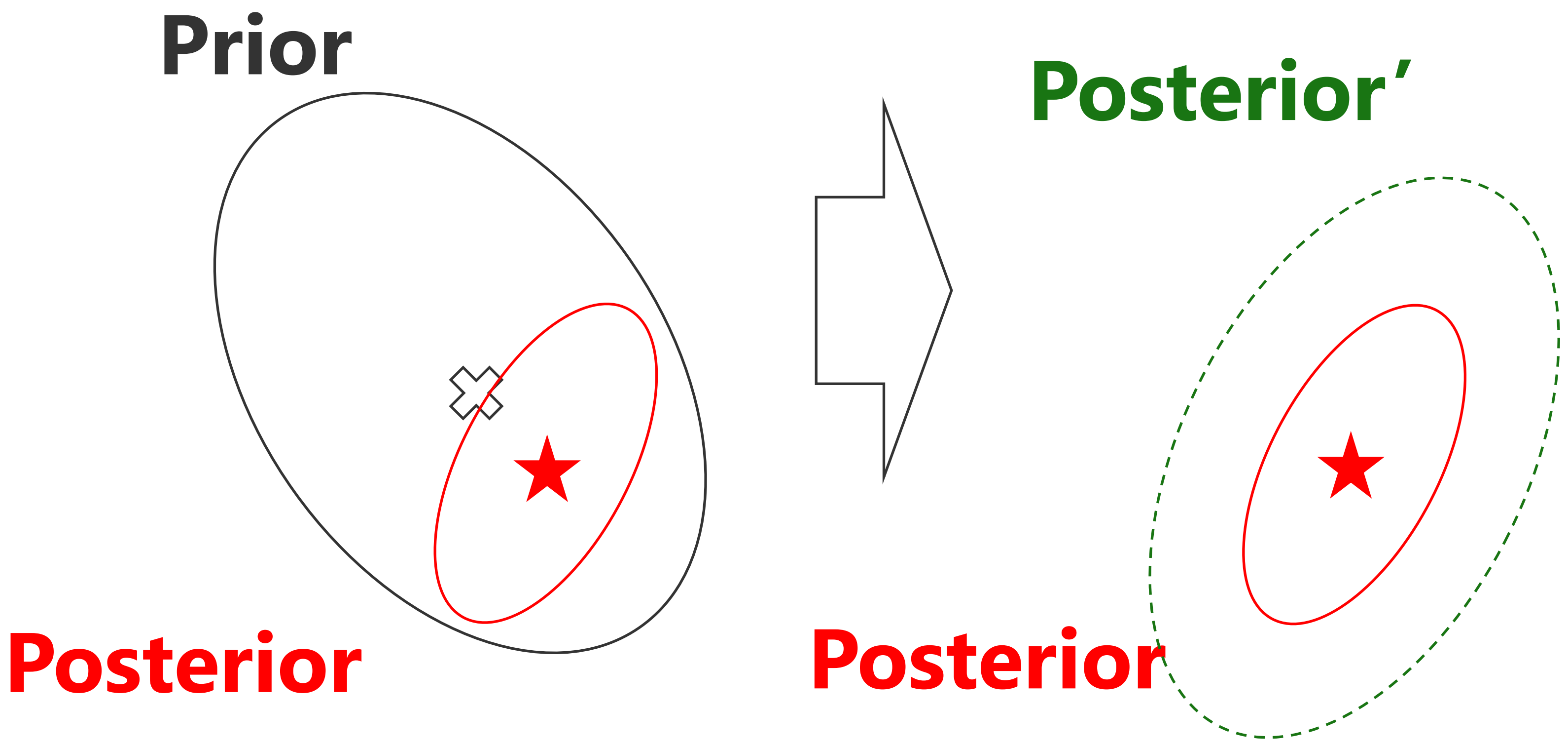
Relaxation to Prior Spread (RTPS)

$$\delta \mathbf{X}^a_{t,inf,i} = \frac{\alpha\sigma_i^b + (1-\alpha)\sigma_i^a}{\sigma_i^a}\,\delta \mathbf{X}^a_{t,i}$$

σ_i: ensemble spread of ith variable

$\xrightarrow{\text{FCST}} \mathbf{P}^b_t$

$\xrightarrow{\text{DA}} \mathbf{P}^a_t \xrightarrow{\text{relaxation}} \mathbf{P}^a_{t,inf} \xrightarrow{\text{FCST}} \mathbf{P}^b_t$



Implementation

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Multiplicative Inflation

Analysis Equations w/o Inflation

subspace spanned by $\mathbf{Z}^b$

mean $$\delta\bar{\mathbf{x}}^a = \mathbf{Z}^b\widetilde{\mathbf{P}}^a\left(\mathbf{Y}^b\right)^T\mathbf{R}^{-1}\mathbf{d}^{o-b} = \mathbf{Z}^b\widetilde{\mathbf{w}}$$

perturbation $$\mathbf{Z}^a = \mathbf{Z}^b(\widetilde{\mathbf{P}}^a)^{1/2} = \mathbf{Z}^b\widetilde{\mathbf{W}}$$

$$\widetilde{\mathbf{P}}^a = \left[(\widetilde{\mathbf{P}}^b)^{-1} + (\mathbf{Y}^b)^T\mathbf{R}^{-1}\mathbf{Y}^b\right]^{-1}$$

$$\widetilde{\mathbf{P}}^b = \mathbf{I}$$

subspace spanned by $\delta\mathbf{X}^b$

$$\delta\bar{\mathbf{x}}^a = \delta\mathbf{X}^b\widehat{\mathbf{P}}^a\left(\delta\mathbf{Y}^b\right)^T\mathbf{R}^{-1}\mathbf{d}^{o-b} = \delta\mathbf{X}^b\widehat{\mathbf{w}}$$

$$\delta\mathbf{X}^a = \delta\mathbf{X}^b((m-1)\widehat{\mathbf{P}}^a)^{1/2} = \delta\mathbf{X}^b\widehat{\mathbf{W}}$$

$$\widehat{\mathbf{P}}^a = \left[(\widehat{\mathbf{P}}^b)^{-1} + (\delta\mathbf{Y}^b)^T\mathbf{R}^{-1}\delta\mathbf{Y}^b\right]^{-1}$$

$$\widehat{\mathbf{P}}^b = \mathbf{I}/(m-1)$$

Multiplicative Inflation

Point: update vector and matrix (**w** and **W**) can be applied for ptb w/o inflation

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Analysis Equations w/ Inflation

$$\mathbf{P}^b_{inf} = \Delta \mathbf{P}^b \Leftrightarrow \delta \mathbf{X}^b_{inf} = \sqrt{\Delta}\delta\mathbf{X}^b$$

subspace spanned by $\sqrt{\Delta}\mathbf{Z}^b$

mean

$$\delta\bar{\mathbf{x}}^a = \sqrt{\Delta}\mathbf{Z}^b\widetilde{\mathbf{P}}^a\left(\sqrt{\Delta}\mathbf{Y}^b\right)^T\mathbf{R}^{-1}\mathbf{d}^{o-b} = \mathbf{Z}^b\Delta\widetilde{\mathbf{P}}^a\left(\mathbf{Y}^b\right)^T\mathbf{R}^{-1}\mathbf{d}^{o-b}$$

ptb

$$\mathbf{Z}^a = \sqrt{\Delta}\mathbf{Z}^b(\widetilde{\mathbf{P}}^a)^{1/2} = \mathbf{Z}^b(\Delta\widetilde{\mathbf{P}}^a)^{1/2} = \mathbf{Z}^b\widetilde{\mathbf{W}}_{inf}$$

$$\widetilde{\mathbf{P}}^b = \mathbf{I}$$

$$\widetilde{\mathbf{P}}^a = \left[(\widetilde{\mathbf{P}}^b)^{-1} + \Delta(\mathbf{Y}^b)^T\mathbf{R}^{-1}\mathbf{Y}^b\right]^{-1}$$

$$\widetilde{\mathbf{P}}^{a\prime} = \Delta\widetilde{\mathbf{P}}^a = \left[(\Delta\widetilde{\mathbf{P}}^b)^{-1} + (\mathbf{Y}^b)^T\mathbf{R}^{-1}\mathbf{Y}^b\right]^{-1} = \left[\mathbf{I}/\Delta + (\mathbf{Y}^b)^T\mathbf{R}^{-1}\mathbf{Y}^b\right]^{-1}$$

subspace spanned by $\sqrt{\Delta}\delta\mathbf{X}^b$

$$\delta\bar{\mathbf{x}}^a = \sqrt{\Delta}\delta\mathbf{X}^b\widehat{\mathbf{P}}^a\left(\sqrt{\Delta}\delta\mathbf{Y}^b\right)^T\mathbf{R}^{-1}\mathbf{d}^{o-b} = \delta\mathbf{X}^b\Delta\widehat{\mathbf{P}}^a\left(\delta\mathbf{Y}^b\right)^T\mathbf{R}^{-1}\mathbf{d}^{o-b}$$

$$\delta\mathbf{X}^a = \sqrt{\Delta}\delta\mathbf{X}^b((m-1)\widehat{\mathbf{P}}^a)^{1/2} = \delta\mathbf{X}^b((m-1)\Delta\widehat{\mathbf{P}}^a)^{1/2} = \delta\mathbf{X}^b\widehat{\mathbf{W}}_{inf}$$

$$\widehat{\mathbf{P}}^b = \mathbf{I}/(m-1)$$

$$\widehat{\mathbf{P}}^a = \left[(\widehat{\mathbf{P}}^b)^{-1} + \Delta(\delta\mathbf{Y}^b)^T\mathbf{R}^{-1}\delta\mathbf{Y}^b\right]^{-1}$$

$$\widehat{\mathbf{P}}^{a\prime} = \Delta\widehat{\mathbf{P}}^a = \left[(\Delta\widehat{\mathbf{P}}^b)^{-1} + (\delta\mathbf{Y}^b)^T\mathbf{R}^{-1}\delta\mathbf{Y}^b\right]^{-1} = \left[(m-1)/\Delta + (\delta\mathbf{Y}^b)^T\mathbf{R}^{-1}\delta\mathbf{Y}^b\right]^{-1}$$

RTPP

RTPP can be implemented within letkf_core

Analysis Equations w/o Inflation

subspace spanned by $\mathbf{Z}^b$

mean $\delta\bar{\mathbf{x}}^a = \mathbf{Z}^b\widetilde{\mathbf{P}}^a\left(\mathbf{Y}^b\right)^T\mathbf{R}^{-1}\mathbf{d}^{o-b} = \mathbf{Z}^b\widetilde{\mathbf{w}}$

perturbation $\mathbf{Z}^a = \mathbf{Z}^b(\widetilde{\mathbf{P}}^a)^{1/2} = \mathbf{Z}^b\widetilde{\mathbf{W}}$

subspace spanned by $\delta\mathbf{X}^b$

$$\delta\bar{\mathbf{x}}^a = \delta\mathbf{X}^b\widehat{\mathbf{P}}^a\left(\delta\mathbf{Y}^b\right)^T\mathbf{R}^{-1}\mathbf{d}^{o-b} = \delta\mathbf{X}^b\widehat{\mathbf{w}}$$

$$\delta\mathbf{X}^a = \delta\mathbf{X}^b((m-1)\widehat{\mathbf{P}}^a)^{1/2} = \delta\mathbf{X}^b\widehat{\mathbf{W}}$$

Analysis Equations w/ RTPP

$$\mathbf{Z}^a_{RTPP} = (1-\alpha)\mathbf{Z}^a + \alpha\mathbf{Z}^b$$

$$\Leftrightarrow \widetilde{\mathbf{W}}_{RTPP} = (1-\alpha)\widetilde{\mathbf{W}} + \alpha\mathbf{I}$$ *RTPP*

$$\delta\mathbf{X}^a_{RTPP} = (1-\alpha)\delta\mathbf{X}^a + \alpha\delta\mathbf{X}^b$$

$$\Leftrightarrow \widehat{\mathbf{W}}_{RTPP} = (1-\alpha)\widehat{\mathbf{W}} + \alpha\mathbf{I}$$ *RTPP*

Analysis Equations w/ RTPP & multiplicative inflation

$$\mathbf{Z}^a_{RTPP} = (1-\alpha)\mathbf{Z}^a + \alpha\sqrt{\Delta}\mathbf{Z}^b$$

where $\mathbf{Z}^a = \mathbf{Z}^b\widetilde{\mathbf{W}}_{inf}$

$$\Leftrightarrow \widetilde{\mathbf{W}}_{RTPP} = (1-\alpha)\widetilde{\mathbf{W}}_{inf} + \alpha\sqrt{\Delta}\mathbf{I}$$ *RTPP*

relaxed to inflated prior perturbation

$$\delta\mathbf{X}^a_{RTPP} = (1-\alpha)\delta\mathbf{X}^a + \alpha\sqrt{\Delta}\delta\mathbf{X}^b$$

where $\delta\mathbf{X}^a = \delta\mathbf{X}^b\widehat{\mathbf{W}}_{inf}$

$$\Leftrightarrow \widehat{\mathbf{W}}_{RTPP} = (1-\alpha)\widehat{\mathbf{W}}_{inf} + \alpha\sqrt{\Delta}\mathbf{I}$$ *RTPP*

(note: https://github.com/gylien/scale-letkf/blob/master/scale/letkf/letkf_tools.f90)

RTPS

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Analysis Equations w/o Inflation

subspace spanned by $\mathbf{Z}^b$

mean $$\delta\bar{\mathbf{x}}^a = \mathbf{Z}^b\widetilde{\mathbf{P}}^a\left(\mathbf{Y}^b\right)^T\mathbf{R}^{-1}\mathbf{d}^{o-b} = \mathbf{Z}^b\widetilde{\mathbf{w}}$$

perturbation $$\mathbf{Z}^a = \mathbf{Z}^b(\widetilde{\mathbf{P}}^a)^{1/2} = \mathbf{Z}^b\widetilde{\mathbf{W}}$$

subspace spanned by $\delta\mathbf{X}^b$

$$\delta\bar{\mathbf{x}}^a = \delta\mathbf{X}^b\widehat{\mathbf{P}}^a\left(\delta\mathbf{Y}^b\right)^T\mathbf{R}^{-1}\mathbf{d}^{o-b} = \delta\mathbf{X}^b\widehat{\mathbf{w}}$$

$$\delta\mathbf{X}^a = \delta\mathbf{X}^b((m-1)\widehat{\mathbf{P}}^a)^{1/2} = \delta\mathbf{X}^b\widehat{\mathbf{W}}$$

Analysis Equations w/ RTPS

$$\mathbf{Z}^a_{inf,i} = (1-\alpha+\alpha\frac{\sigma_i^b}{\sigma_i^a})\mathbf{Z}^a_i$$

$$\frac{\sigma_i^b}{\sigma_i^a} = \sqrt{\frac{\mathbf{Z}_i^b\left(\mathbf{Z}_i^b\right)^T}{\mathbf{Z}_i^a\left(\mathbf{Z}_i^a\right)^T}} = \sqrt{\frac{\mathbf{Z}_i^b\left(\mathbf{Z}_i^b\right)^T}{\mathbf{Z}_i^b\widetilde{\mathbf{P}}^a\left(\mathbf{Z}_i^b\right)^T}}$$

$$\widetilde{\mathbf{W}}_{RTPS} = \left(1-\alpha+\alpha\sqrt{\frac{\mathbf{Z}_i^b\left(\mathbf{Z}_i^b\right)^T}{\mathbf{Z}_i^b\widetilde{\mathbf{P}}^a\left(\mathbf{Z}_i^b\right)^T}}\right)\widetilde{\mathbf{W}}$$

$$\delta\mathbf{X}^a_{inf,i} = (1-\alpha+\alpha\frac{\sigma_i^b}{\sigma_i^a})\delta\mathbf{X}^a_i$$

$$\frac{\sigma_i^b}{\sigma_i^a} = \sqrt{\frac{\delta\mathbf{X}_i^b\left(\delta\mathbf{X}_i^b\right)^T}{\delta\mathbf{X}_i^a\left(\delta\mathbf{X}_i^a\right)^T}} = \sqrt{\frac{\delta\mathbf{X}_i^b\left(\delta\mathbf{X}_i^b\right)^T}{\delta\mathbf{X}_i^b\widehat{\mathbf{P}}^a\left(\delta\mathbf{X}_i^b\right)^T}}$$

$$\widehat{\mathbf{W}}_{RTPS} = \left(1-\alpha+\alpha\sqrt{\frac{\delta\mathbf{X}_i^b\left(\delta\mathbf{X}_i^b\right)^T}{\delta\mathbf{X}_i^b\widehat{\mathbf{P}}^a\left(\delta\mathbf{X}_i^b\right)^T}}\right)\widehat{\mathbf{W}}$$

RTPS cannot be implemented within letkf_core because of model vars are necessary

RTPS (cont'd)

Analysis Equations w/ RTPS & multiplicative inflation

$$\mathbf{Z}^a_{inf,i} = \left(1-\alpha+\alpha\frac{\sqrt{\Delta}\sigma_i^b}{\sigma_i^a}\right)\mathbf{Z}_i^a$$

$$\delta\mathbf{X}^a_{inf,i} = \left(1-\alpha+\alpha\frac{\sqrt{\Delta}\sigma_i^b}{\sigma_i^a}\right)\delta\mathbf{X}_i^a$$

$$\frac{\sqrt{\Delta}\sigma_i^b}{\sigma_i^a} = \sqrt{\frac{\Delta\mathbf{Z}_i^b\left(\mathbf{Z}_i^b\right)^T}{\mathbf{Z}_i^b\widetilde{\mathbf{W}}_{inf}\left(\mathbf{Z}_i^b\widetilde{\mathbf{W}}_{inf}\right)^T}}$$

$$\frac{\sqrt{\Delta}\sigma_i^b}{\sigma_i^a} = \sqrt{\frac{\Delta\delta\mathbf{X}_i^b\left(\delta\mathbf{X}_i^b\right)^T}{\delta\mathbf{X}_i^b\widehat{\mathbf{W}}_{inf}\left(\delta\mathbf{X}_i^b\widehat{\mathbf{W}}_{inf}\right)^T}}$$

$$\widetilde{\mathbf{W}}_{inf}\left(\widetilde{\mathbf{W}}_{inf}\right)^T = \Delta\widetilde{\mathbf{P}}^a = \widetilde{\mathbf{P}}^{a\prime}$$

$$\widehat{\mathbf{W}}_{inf}\left(\widehat{\mathbf{W}}_{inf}\right)^T = \Delta\widehat{\mathbf{P}}^a = \widehat{\mathbf{P}}^{a\prime}$$

$$\widetilde{\mathbf{W}}_{RTPS} = \left(1-\alpha+\alpha\sqrt{\frac{\mathbf{Z}_i^b\left(\mathbf{Z}_i^b\right)^T}{\mathbf{Z}_i^b\widetilde{\mathbf{P}}^{a\prime}\left(\mathbf{Z}_i^b\right)^T}}\right)\widetilde{\mathbf{W}}_{inf}$$

$$\widehat{\mathbf{W}}_{RTPS} = \left(1-\alpha+\alpha\sqrt{\frac{\Delta\delta\mathbf{X}_i^b\left(\delta\mathbf{X}_i^b\right)^T}{\delta\mathbf{X}_i^b\widehat{\mathbf{P}}^{a\prime}\left(\delta\mathbf{X}_i^b\right)^T}}\right)\widehat{\mathbf{W}}_{inf}$$

(note: https://github.com/gylien/scale-letkf/blob/master/scale/letkf/letkf_tools.f90)

Comparison of Inflation Methods

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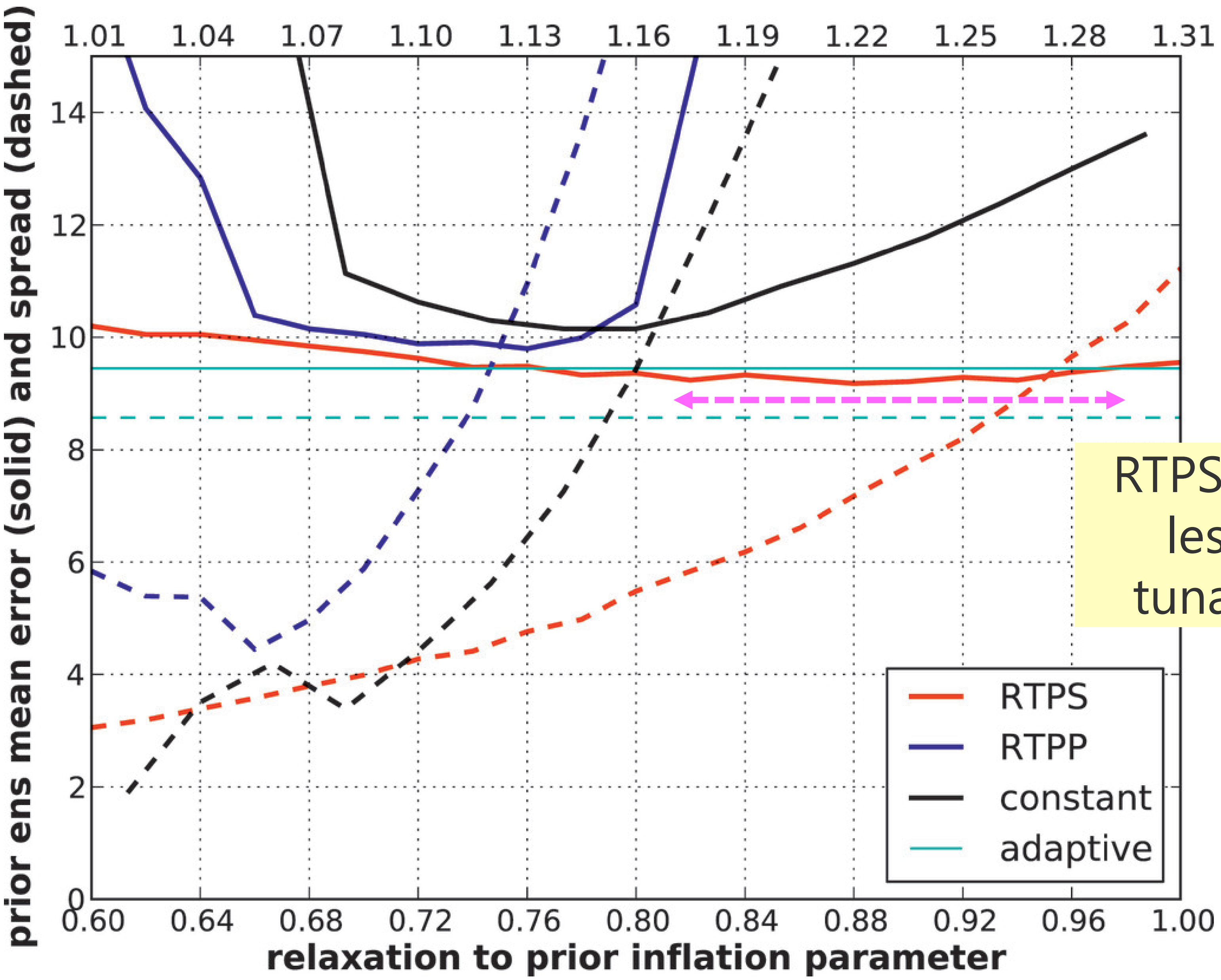
Sensitivity to tunable parameters

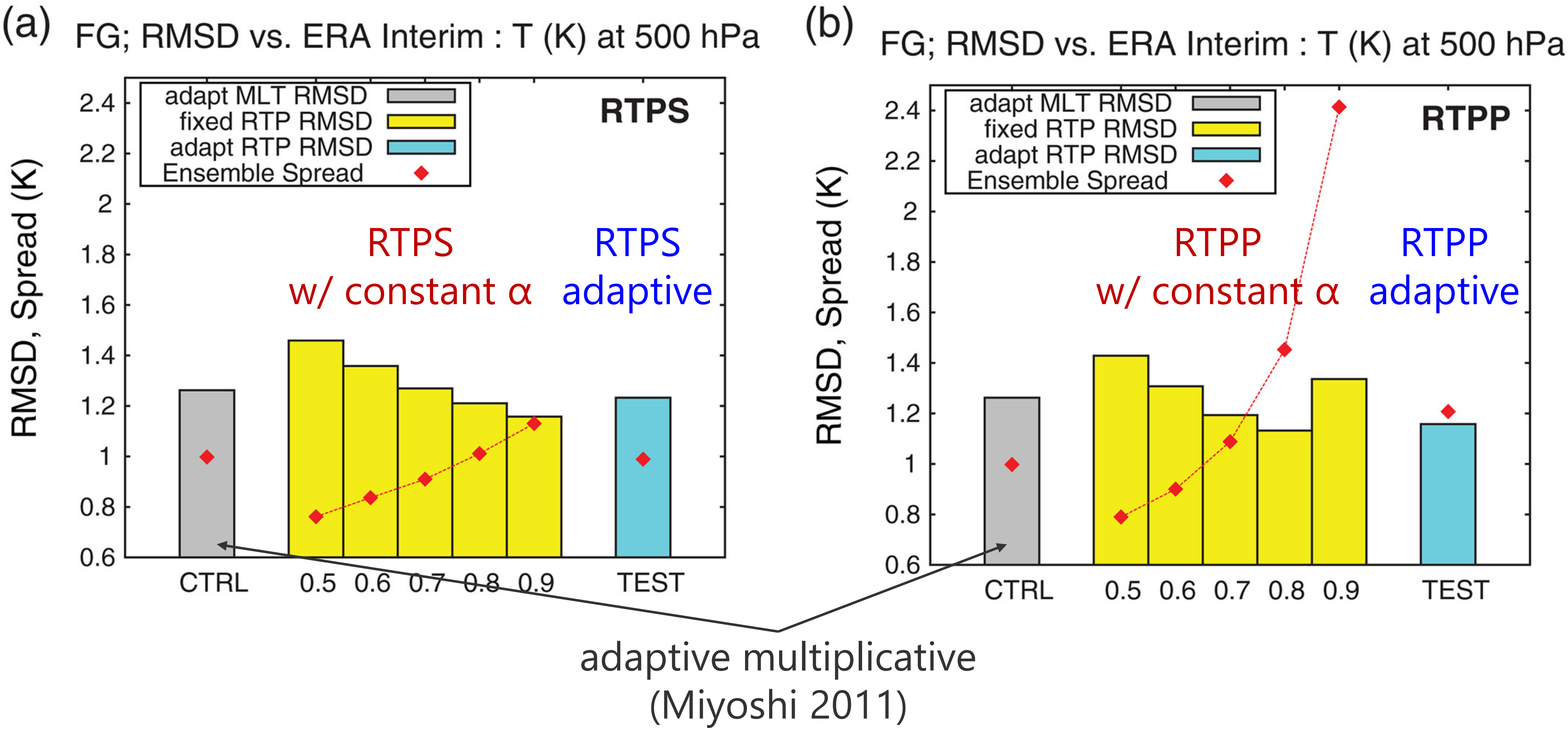
Whitaker and Hamill (2012; MWR)



RTPS is known to be less sensitive to tunable parameter

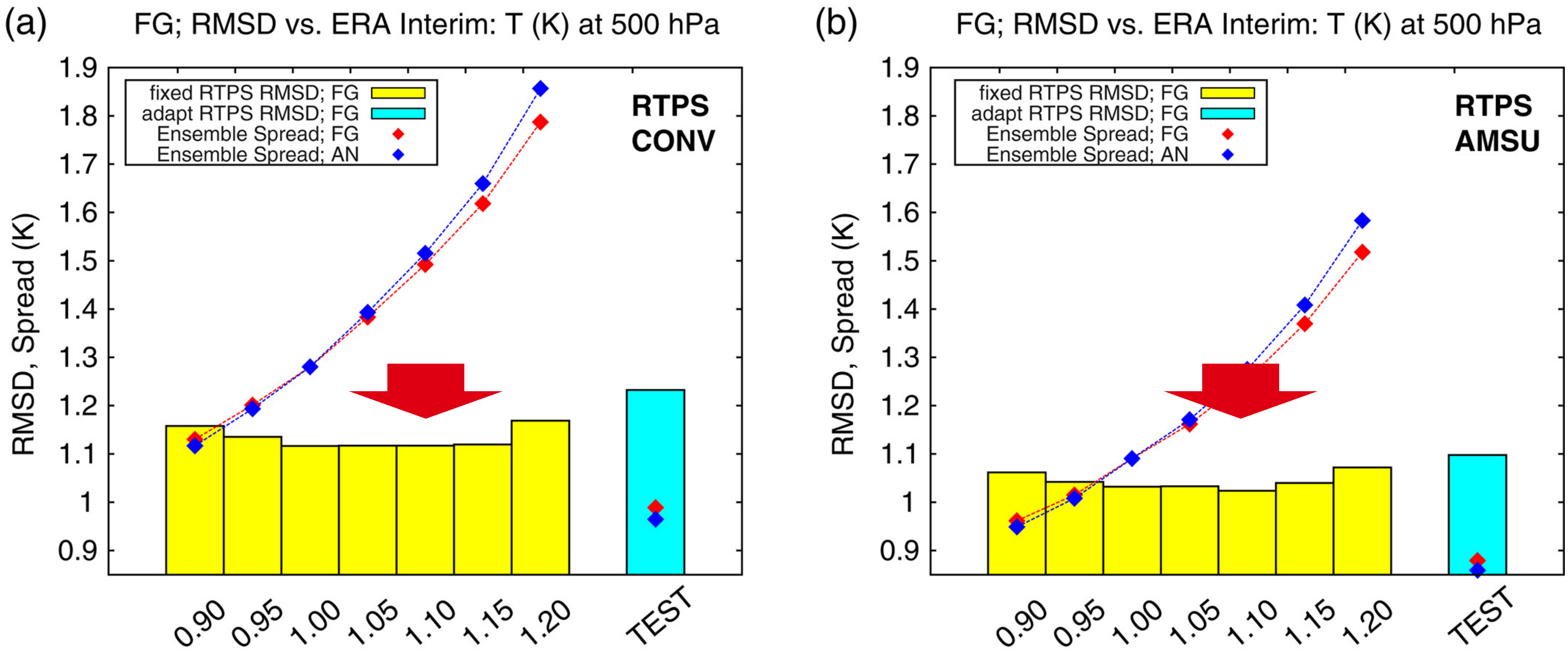
A case with NICAM-LETKF

Kotsuki et al. (2017; QJRMS)



A case with NICAM-LETKF

Kotsuki et al. (2017; QJRMS)



The optimal relaxation parameter can be α > 1.00,
meaning that posterior spread is than prior spread

Why does EnKF suffer from analysis overconfidence?

Quarterly Journal of the Royal Meteorological Society

RESEARCH ARTICLE | Full Access

Why does EnKF suffer from analysis overconfidence? An insight into exploiting the ever-increasing volume of observations

Daisuke Hotta, Yoichiro Ota

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Thank you for your attention!

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Further information is available at
https://kotsuki-lab.com/

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