Data Assimilation
- O01. FSO & EFSO -

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Environmental Prediction Science Laboratory

Observation Diagnosis

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Observation Space Analysis

- Innovation statistics
- Degrees of freedom to signals

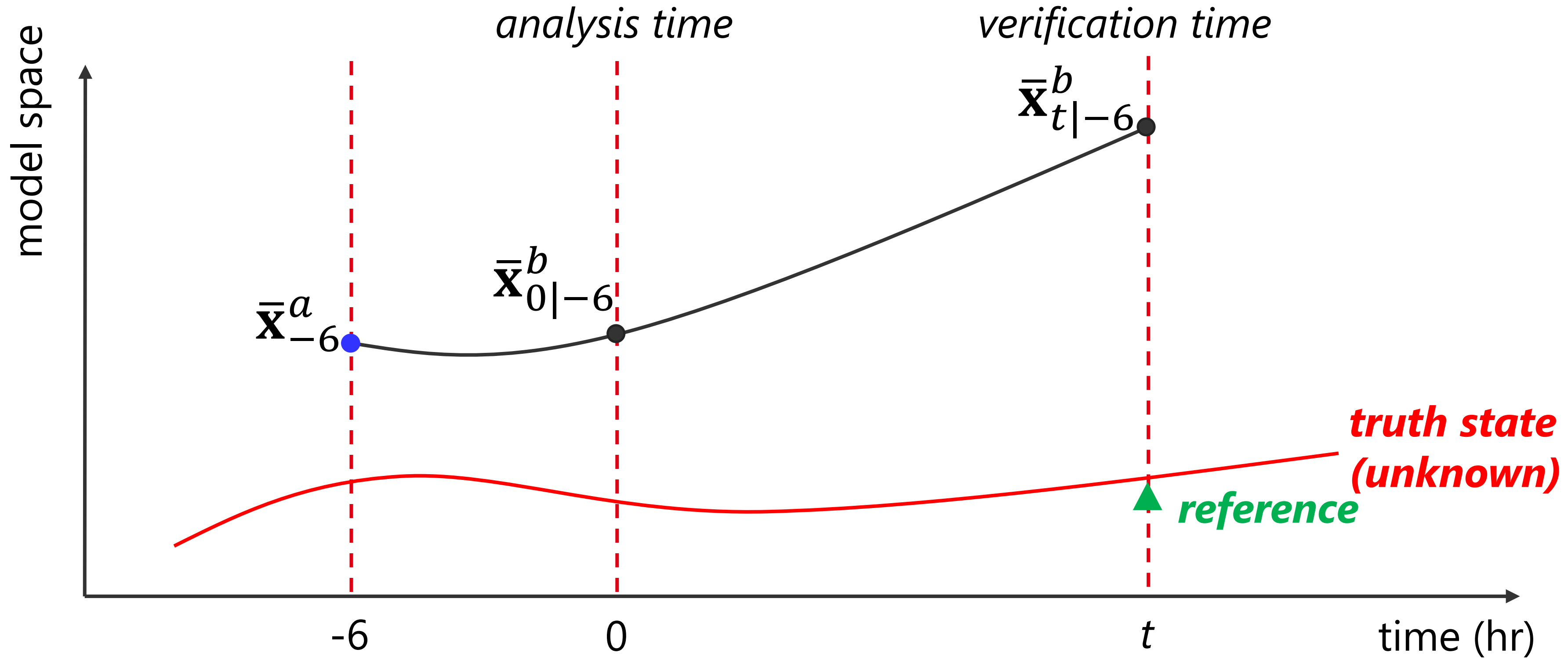
Forecast Sensitivity

		Forecast Sensitivity to Observation (FSO)	
		Adjoint	Ensemble
DA (iteration)	iterative	Langland and Baker (2004)	Buehner et al. (2018)
	deterministic	N/A	Kalnay et al. (2012)

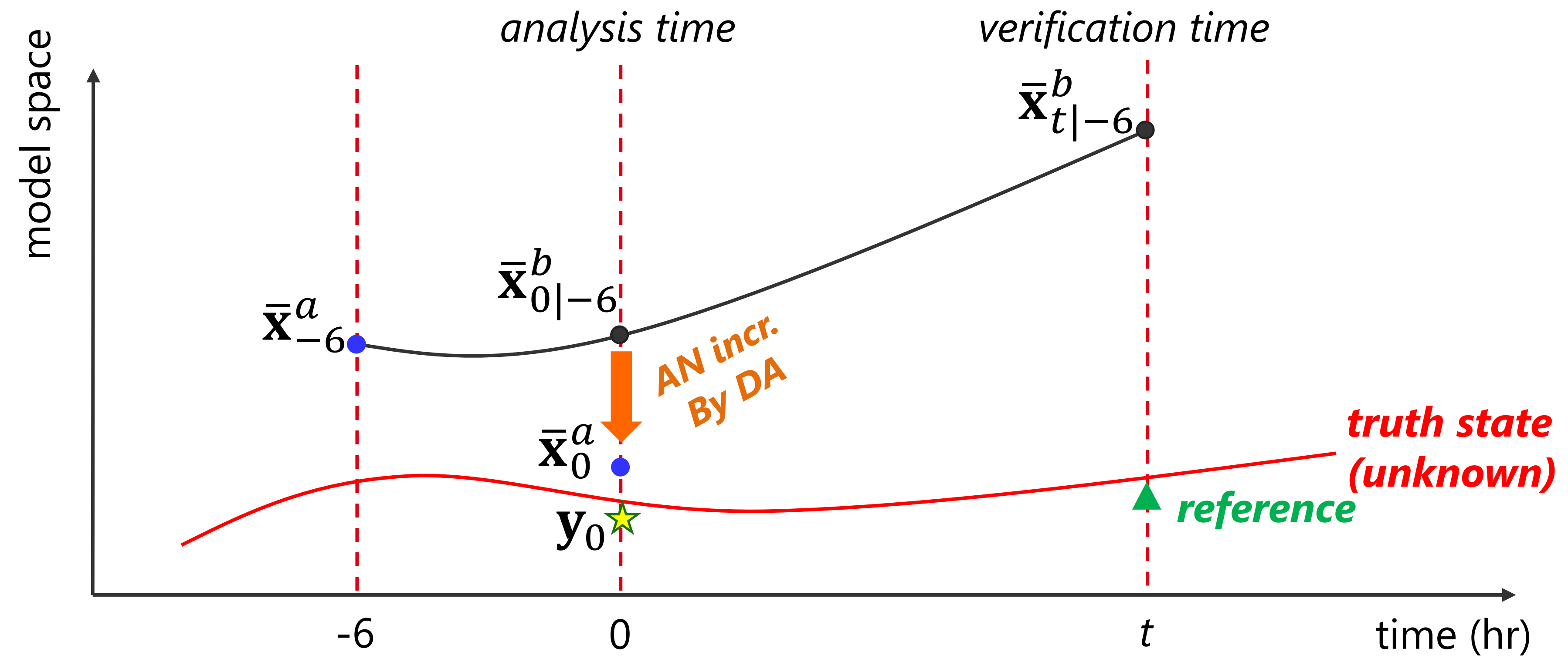
		Forecast Sensitivity to **R** (FSR)	
		Adjoint	Ensemble
DA (iteration)	Adjoint	Daescu (2008)	N/A
	Ensemble	N/A	Hotta et al. (2017)

Forecast Sensitivity to Observation (FSO)

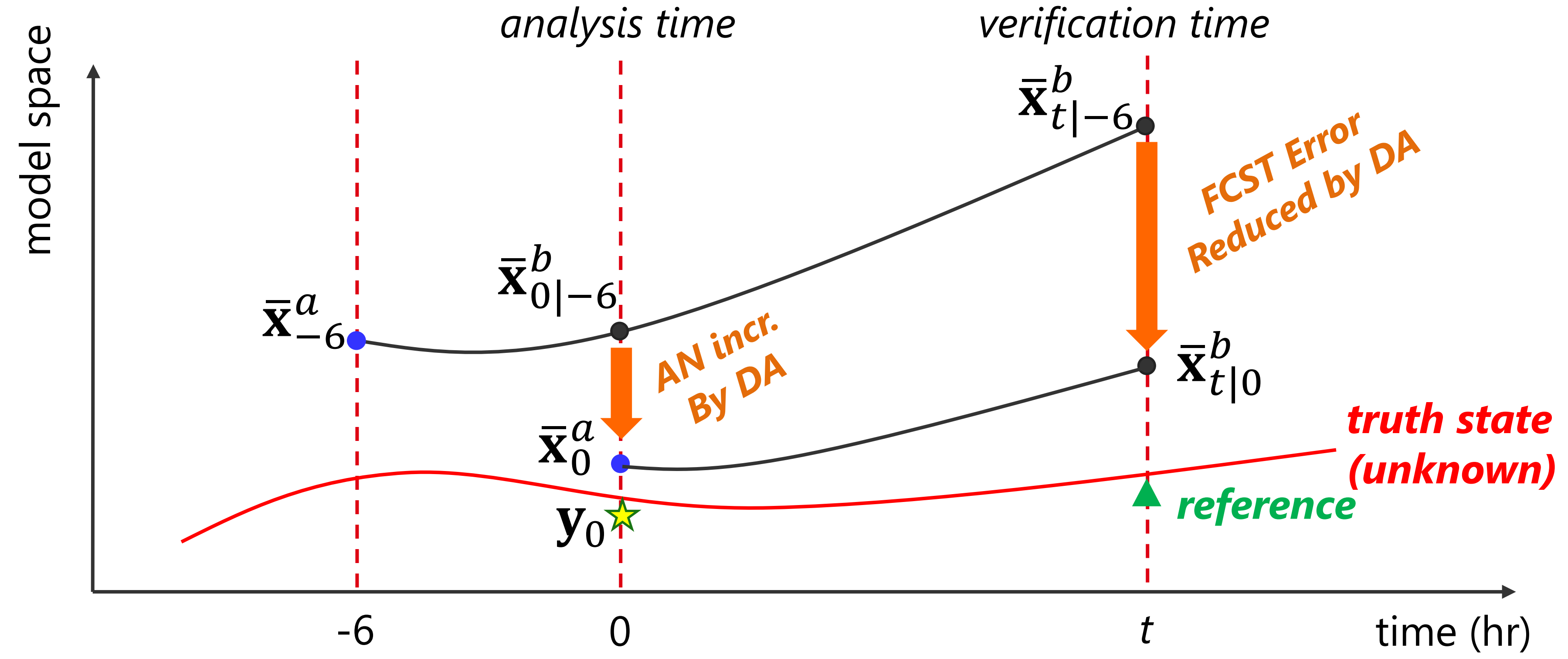
FSO: Forecast Sensitivity to Obs



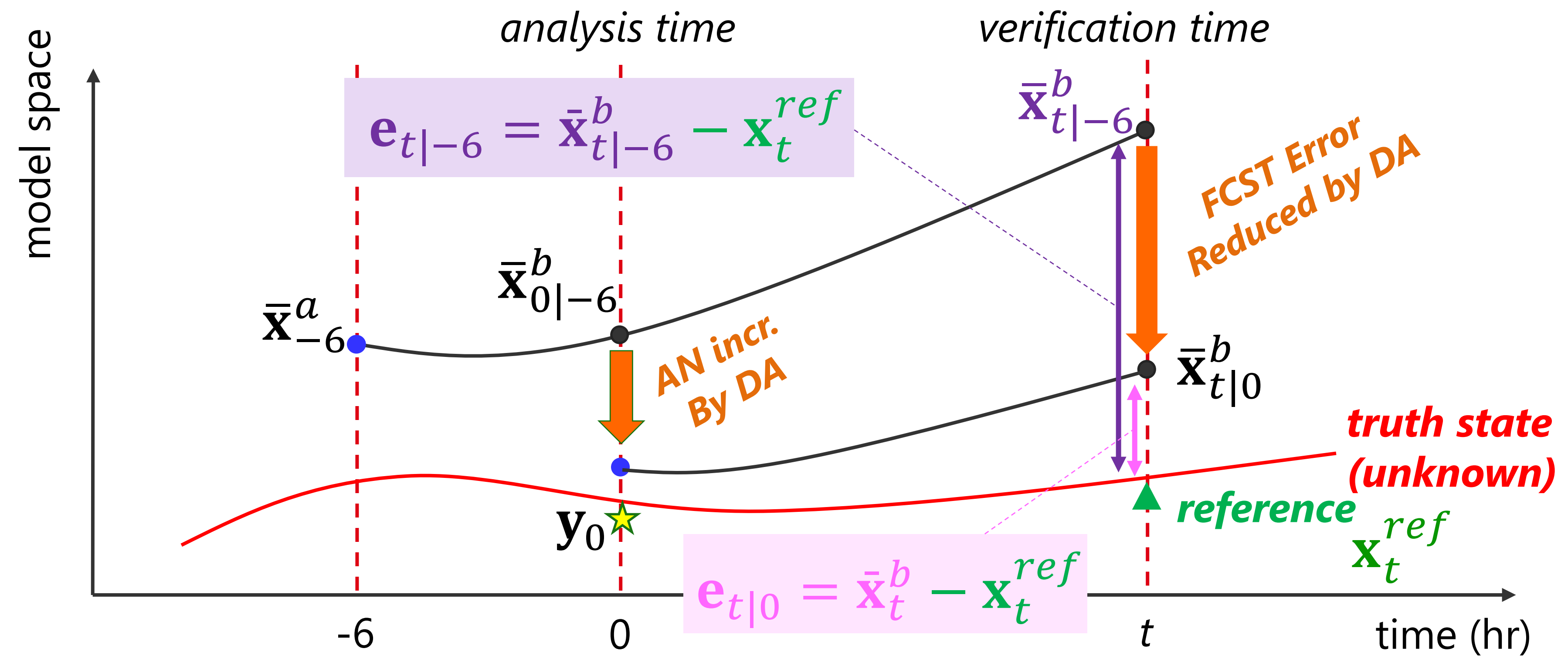
FSO: Forecast Sensitivity to Obs



FSO: Forecast Sensitivity to Obs



FSO: Forecast Sensitivity to Obs



The difference b/w $\mathbf{e}_{t|0}$ and $\mathbf{e}_{t|-6}$ depends only on the obs DA'ed at t=0.

FCST Error Reduction $$J = \Delta e^2 = e^2_{t|0} - e^2_{t|-6} = \mathbf{e}^T_{t|0} C \mathbf{e}_{t|0} - \mathbf{e}^T_{t|-6} C \mathbf{e}_{t|-6}$$

Error Reduction w.r.t. Obs $$\frac{\partial J}{\partial \mathbf{y}} \in \mathbb{R}^p$$

C : square norms
(e.g. L2 for L63/L96, dry/moist energy norm for NWP)

FCST Sensitivity to Obs (FSO)

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$$J = \Delta e^2 = \mathbf{e}_{t|0}^T C \mathbf{e}_{t|0} - \mathbf{e}_{t|0}^T C \mathbf{e}_{t|-6}$$

$$= \left(\mathbf{e}_{t|0} - \mathbf{e}_{t|-6}\right)^T C\left(\mathbf{e}_{t|0} + \mathbf{e}_{t|-6}\right)$$

$$= \left(\bar{\mathbf{x}}_{t|0}^b - \bar{\mathbf{x}}_{t|-6}^b\right)^T C\left(\mathbf{e}_{t|0} + \mathbf{e}_{t|-6}\right)$$

$$\approx \left[\mathbf{M}\left(\bar{\mathbf{x}}_0^a - \bar{\mathbf{x}}_{0|-6}^b\right)\right]^T C\left(\mathbf{e}_{t|0} + \mathbf{e}_{t|-6}\right)$$

$$= \left(\mathbf{M}\mathbf{K}\delta\mathbf{y}_o\right)^T C\left(\mathbf{e}_{t|0} + \mathbf{e}_{t|-6}\right)$$

$$= \delta\mathbf{y}_o^T \mathbf{K}^T \mathbf{M}^T C\left(\mathbf{e}_{t|0} + \mathbf{e}_{t|-6}\right)$$

liner error growth approx.

$\mathbf{K} \in \mathbb{R}^{n\times p}, \mathbf{M} \in \mathbb{R}^{n\times n}$

too large memory

needs adjoint ($\mathbf{K}^T$ & $\mathbf{M}^T$)

$$\bar{\mathbf{x}}_0^a - \bar{\mathbf{x}}_{0|-6}^b = \mathbf{K}\left(\mathbf{y} - H\left(\bar{\mathbf{x}}_{0|-6}^b\right)\right)$$

$$\bar{\mathbf{x}}_0^a - \bar{\mathbf{x}}_{0|-6}^b = \mathbf{K}\delta\mathbf{y}_o$$

$\delta\mathbf{y}_o \in \mathbb{R}^p$

$\mathbf{u} = \mathbf{K}^T \mathbf{M}^T C\left(\mathbf{e}_{t|0} + \mathbf{e}_{t|-6}\right) \in \mathbb{R}^p$

$$\therefore \frac{\partial J}{\partial \mathbf{y}} \approx \mathbf{K}^T \mathbf{M}^T C\left(\mathbf{e}_{t|0} + \mathbf{e}_{t|-6}\right)$$

$$J = \sum_{i=1}^{p} \delta y_{o,i} u_i$$

i.e., impact of ith obs is $\delta y_{o,i} u_i$

Ensemble FSO

$$J = \Delta e^2 = \mathbf{e}_{t|0}^T C \mathbf{e}_{t|0} - \mathbf{e}_{t|0}^T C \mathbf{e}_{t|-6}$$

$$= (\mathbf{MK}\delta\mathbf{y}_o)^T C\left(\mathbf{e}_{t|0} + \mathbf{e}_{t|-6}\right)$$

$$\approx \delta\mathbf{y}_0^T \mathbf{R}^{-1}\mathbf{Y}_0^a \mathbf{X}_{t|0}^{bT} C\left(\mathbf{e}_{t|0} + \mathbf{e}_{t|-6}\right)/(m-1)$$

$\delta\mathbf{y}_o \in \mathbb{R}^p$

$\mathbf{u} = \mathbf{K}^T\mathbf{M}^T C\left(\mathbf{e}_{t|0} + \mathbf{e}_{t|-6}\right) \in \mathbb{R}^p$

$$J = \sum_{i=1}^{p} \delta y_{o,i} u_i$$

i.e., impact of ith obs is $\delta y_{o,i} u_i$

$$M\mathbf{K}\delta\mathbf{y}_0 = M\mathbf{A}\mathbf{H}^T\mathbf{R}^{-1}\delta\mathbf{y}_0$$

ensemble approx. $\mathbf{A} = \frac{1}{m-1}\mathbf{X}_0^a(\mathbf{X}_0^a)^T$

$$\approx M\mathbf{X}_0^a(\mathbf{H}\mathbf{X}_0^a)^T\mathbf{R}^{-1}\delta\mathbf{y}_0/(m-1)$$

$$= \mathbf{X}_{t|0}^b {\mathbf{Y}_0^a}^T \mathbf{R}^{-1}\delta\mathbf{y}_0/(m-1)$$

$\delta\mathbf{Y}_0^a \equiv \mathbf{H}\delta\mathbf{X}_0^a$

Experiments
w/ Lorenz96

Kotsuki, S., Greybush, S., and Miyoshi, T. (2017):
Can we optimize the assimilation order in the serial ensemble Kalman filter?
A study with the Lorenz-96 model.
Mon. Wea. Rev., 145, 4977-4995. doi: 10.1175/MWR-D-17-0094.1

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Experimental Setting

- Serial EnSRF (Whitaker and Hamill 2001)
  - Ensemble size : 8
  - # of observations : 40
  - Adaptive multiplicative inflation (Miyoshi 2011)

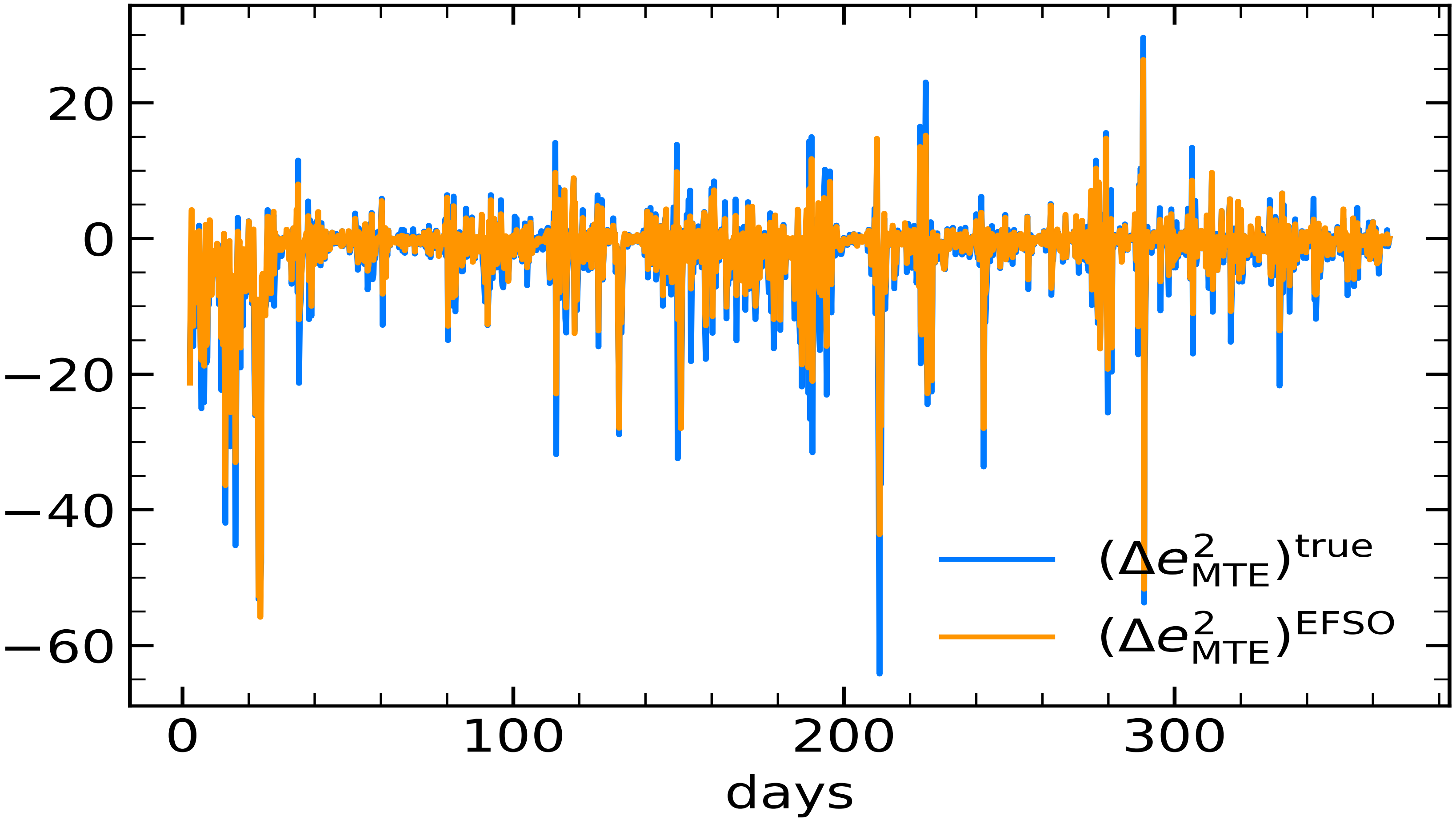
- Assimilation order
1. **Randomly-determined**
2. **From better to worse obs based on EFSO**

cf. Kotsuki et al. (2017, MWR)

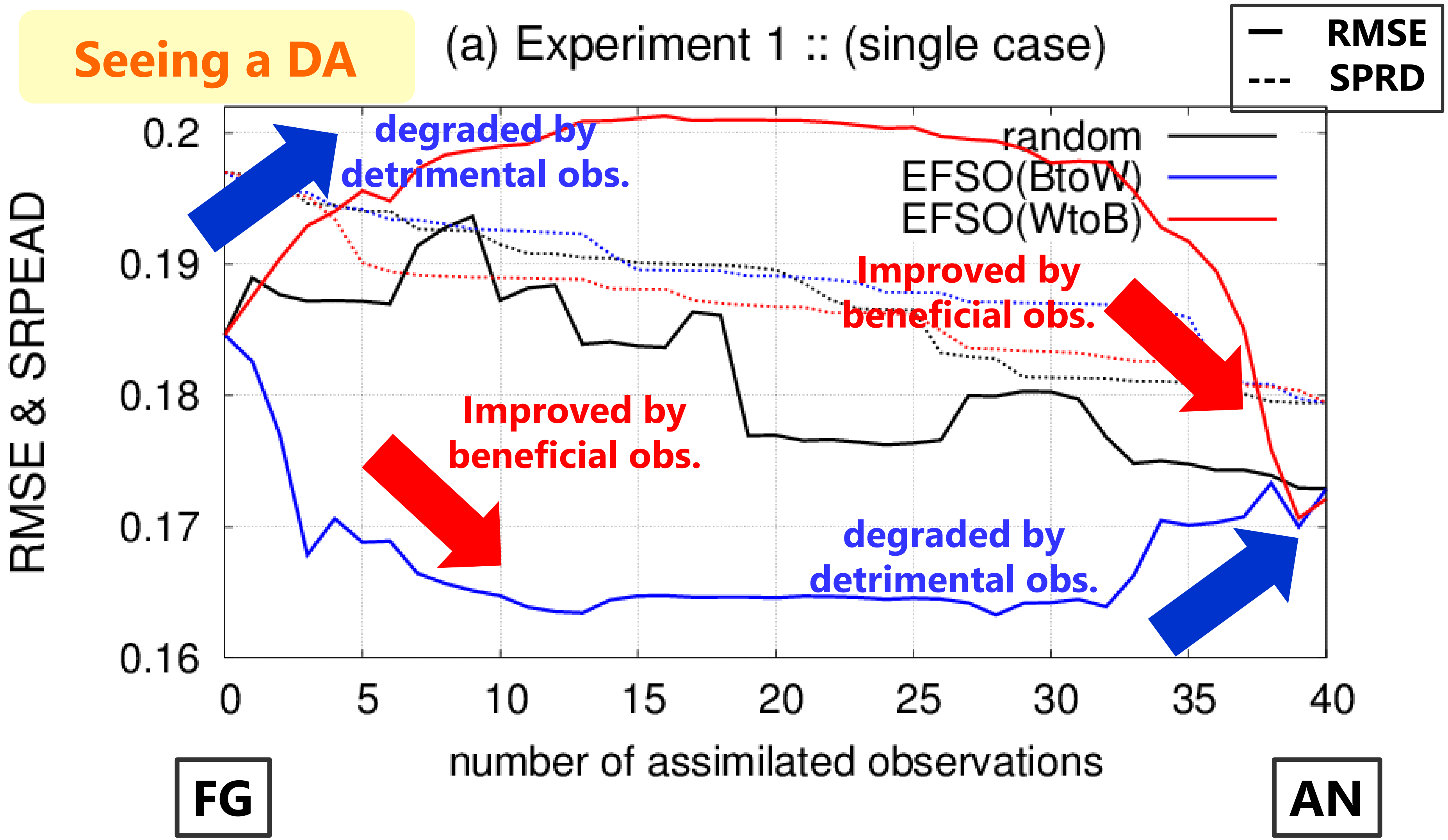
Debug

$$\Delta e^2_{true} = \mathbf{e}^T_{t|0} C \mathbf{e}_{t|0} - \mathbf{e}^T_{t|0} C \mathbf{e}_{t|-6}$$

$$\Delta e^2_{EFSO} = \delta \mathbf{y}^T_0 \mathbf{R}^{-1} \mathbf{Y}^a_0 \mathbf{X}^{bT}_{t|0} C \left(\mathbf{e}_{t|0} + \mathbf{e}_{t|-6}\right)/(m-1)$$



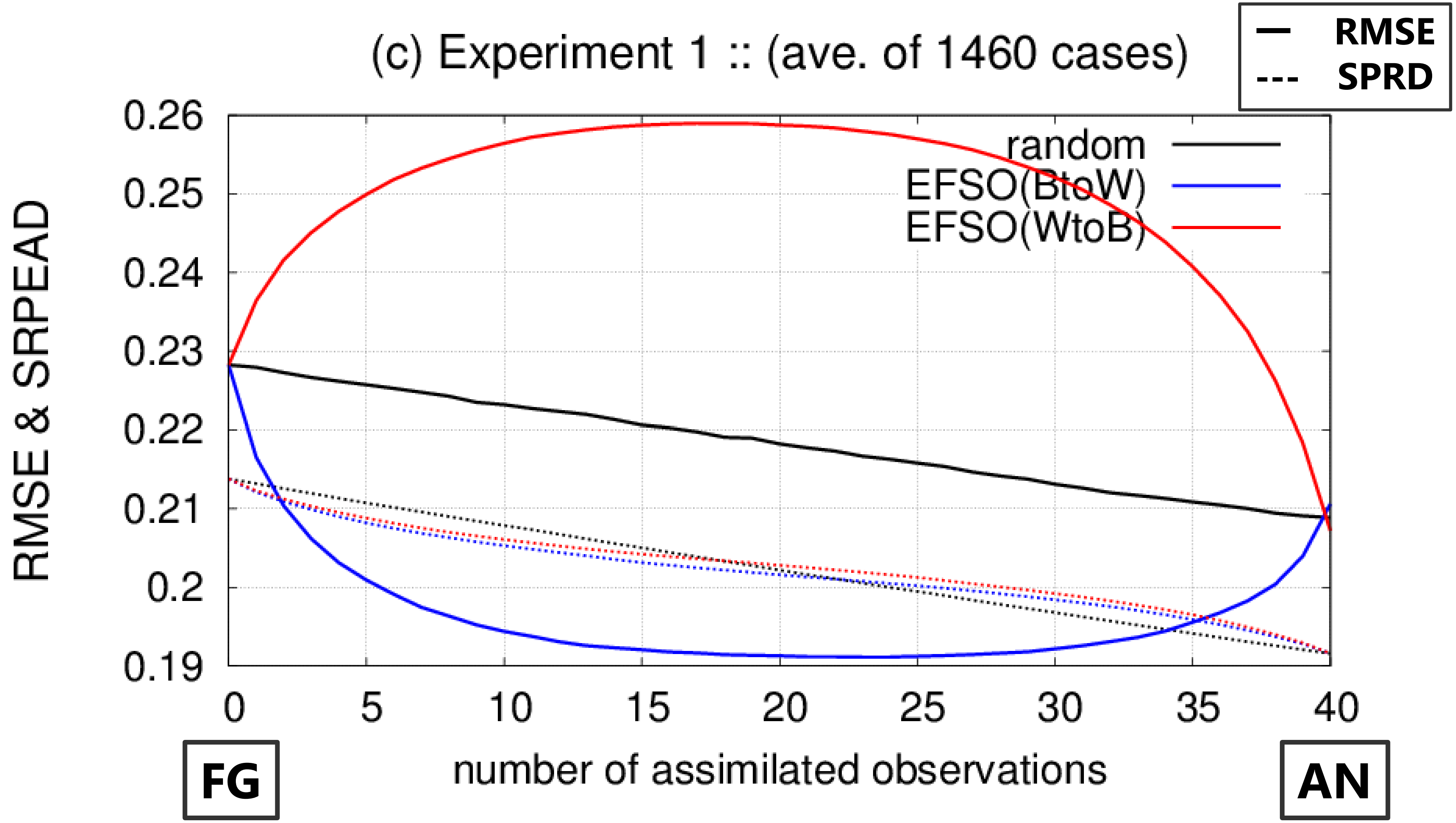
A case w/ serial EnSRF



—: random order

—: EFSO better → worse obs.

Ave of 1460 cases

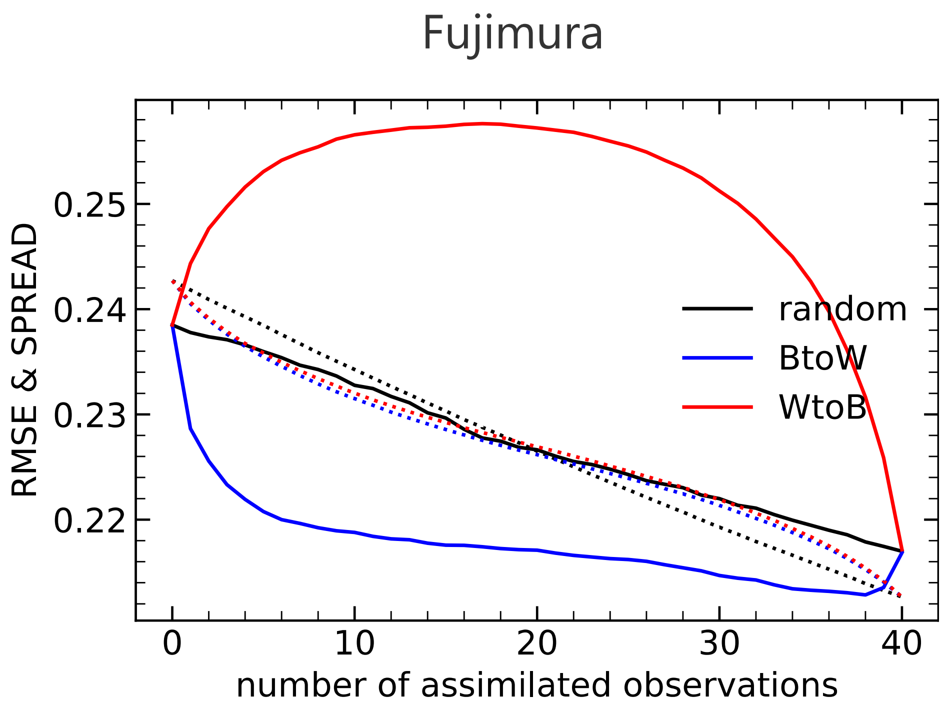


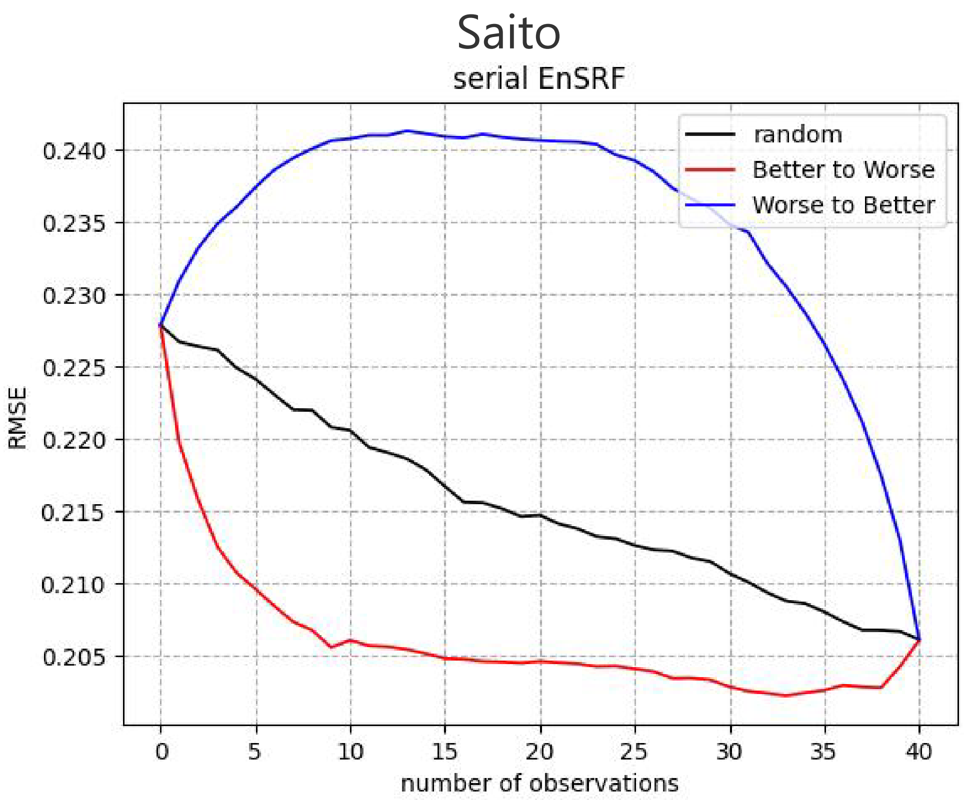
–: random order

–: EFSO better → worse obs.

–: EFSO worse → better obs.

Replication by students





Experiments
w/ NICAM-LETKF

Kotsuki, S.*, Kurosawa, K., and Miyoshi, T. (2019):
On the Properties of Ensemble Forecast Sensitivity to Observations.
Q. J. R. Meteorol. Soc., 145, 1897-1914. doi: 10.1002/qj.3534

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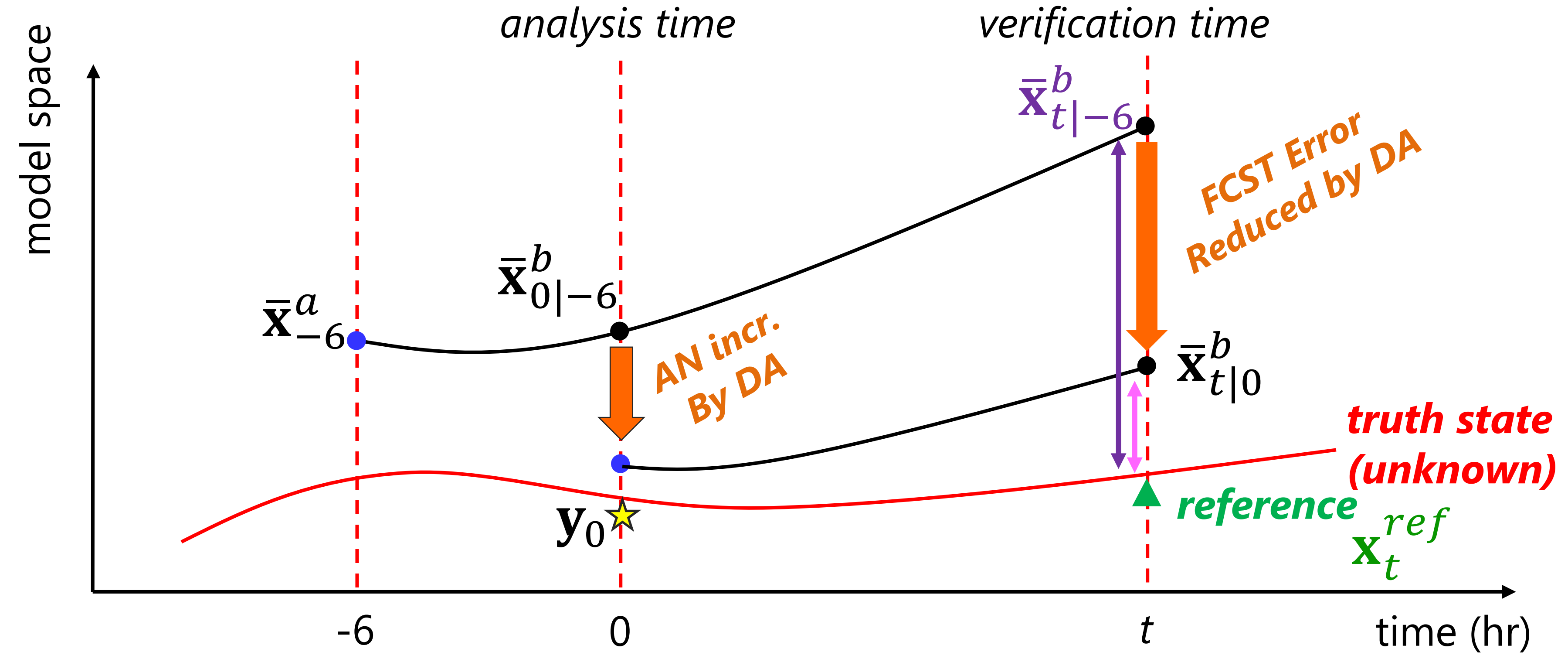
Reference & Norms



Moist Total Energy for Error Norm

Verified against subsequent anl., or anl from other center

$$\Delta e^2_{MTE} = (\mathbf{e}^T_{t|0} C \mathbf{e}_{t|0} - \mathbf{e}^T_{t|-6} C \mathbf{e}_{t|-6})/2$$

$$\mathbf{e}_t = \bar{\mathbf{x}}^b_t - \mathbf{x}^{ref}_t$$

Normalized Obs. Departure for Error Norm

Verified against future obs

$$\Delta e^2_{NOD} = (\mathbf{d}^T_{t|0} \mathbf{R}^{-1}_t \mathbf{d}_{t|0} - \mathbf{d}^T_{t|-6} \mathbf{R}^{-1}_t \mathbf{d}_{t|-6})/p$$

$$\mathbf{d}_t = \mathbf{y}^o_t - H(\mathbf{x}^b_t)$$

Ensemble FSO

Moist Total Energy for Error Norm *(Ota et al. 2013)*

$$\Delta e^2_{MTE} = (\mathbf{e}^T_{t|0} C \mathbf{e}_{t|0} - \mathbf{e}^T_{t|-6} C \mathbf{e}_{t|-6})/2 \qquad \mathbf{e}_t = \bar{\mathbf{x}}^b_t - \mathbf{x}^{ref}_t$$

$$\approx \frac{1}{2}\frac{1}{m-1} \delta\mathbf{y}^T_0 \mathbf{R}^{-1} \mathbf{Y}^a_0 \mathbf{X}^{bT}_{t|0} C \left(\mathbf{e}_{t|0} + \mathbf{e}_{t|-6}\right)$$

obs-minus-FG | AN ptb in obs space | FCST ptb

Normalized Obs. Departure for Error Norm *(Sommer and Weissmann 2016)*

$$\Delta e^2_{NOD} = (\mathbf{d}^T_{t|0} \mathbf{R}^{-1}_t \mathbf{d}_{t|0} - \mathbf{d}^T_{t|-6} \mathbf{R}^{-1}_t \mathbf{d}_{t|-6})/p \qquad \mathbf{d}_t = \mathbf{y}^o_t - H(\mathbf{x}^b_t)$$

$$\approx -\frac{1}{p}\frac{1}{m-1} \delta\mathbf{y}^T_0 \mathbf{R}^{-1} \mathbf{Y}^a_0 \mathbf{Y}^{bT}_{t|0} \mathbf{R}^{-1}_t \left(\mathbf{d}_{t|0} + \mathbf{d}_{t|-6}\right)$$

obs-minus-FG | AN ptb in obs space | FCST ptb

Ensemble FSO (proof of NOD)

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***N*ormalized *O*bs. *D*eparture for Error Norm** (Sommer and Weissmann 2016)

$$\Delta e^2_{NOD} = (\mathbf{d}^T_{t|0}\mathbf{R}^{-1}_t\mathbf{d}_{t|0} - \mathbf{d}^T_{t|-6}\mathbf{R}^{-1}_t\mathbf{d}_{t|-6})/p$$

$$= \left(\mathbf{d}_{t|0} - \mathbf{d}_{t|-6}\right)^T\mathbf{R}^{-1}_t\left(\mathbf{d}_{t|0} + \mathbf{d}_{t|-6}\right)$$

$$\mathbf{d}_{t|0} - \mathbf{d}_{t|-6} = H(\mathbf{x}^b_{t|0}) + H(\mathbf{x}^b_{t|-6})$$

$$= \left(-H(\mathbf{x}^b_{t|0}) + H(\mathbf{x}^b_{t|-6})\right)^T\mathbf{R}^{-1}_t\left(\mathbf{d}_{t|0} + \mathbf{d}_{t|-6}\right)$$

$$\approx -\left[\mathbf{HM}\left(\bar{\mathbf{x}}^a_0 - \bar{\mathbf{x}}^b_{0|-6}\right)\right]^T\mathbf{R}^{-1}_t\left(\mathbf{d}_{t|0} + \mathbf{d}_{t|-6}\right)$$

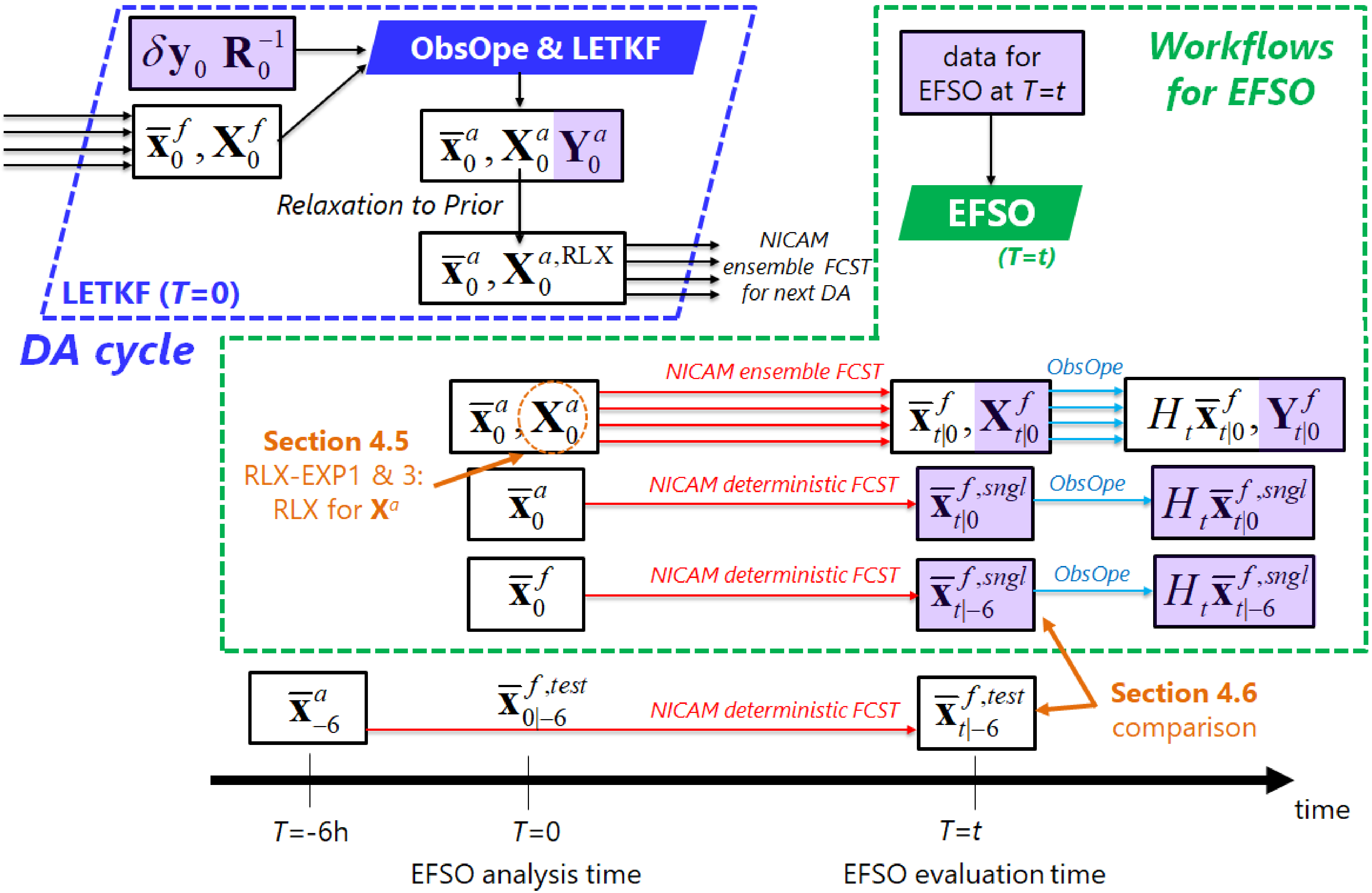
$$= -(\mathbf{HMK}\delta\mathbf{y}_o)^T\mathbf{R}^{-1}_t\left(\mathbf{d}_{t|0} + \mathbf{d}_{t|-6}\right)$$

$$= -\delta\mathbf{y}^T_o\mathbf{K}^T\mathbf{M}^T\mathbf{H}^T\mathbf{R}^{-1}_t\left(\mathbf{d}_{t|0} + \mathbf{d}_{t|-6}\right)$$

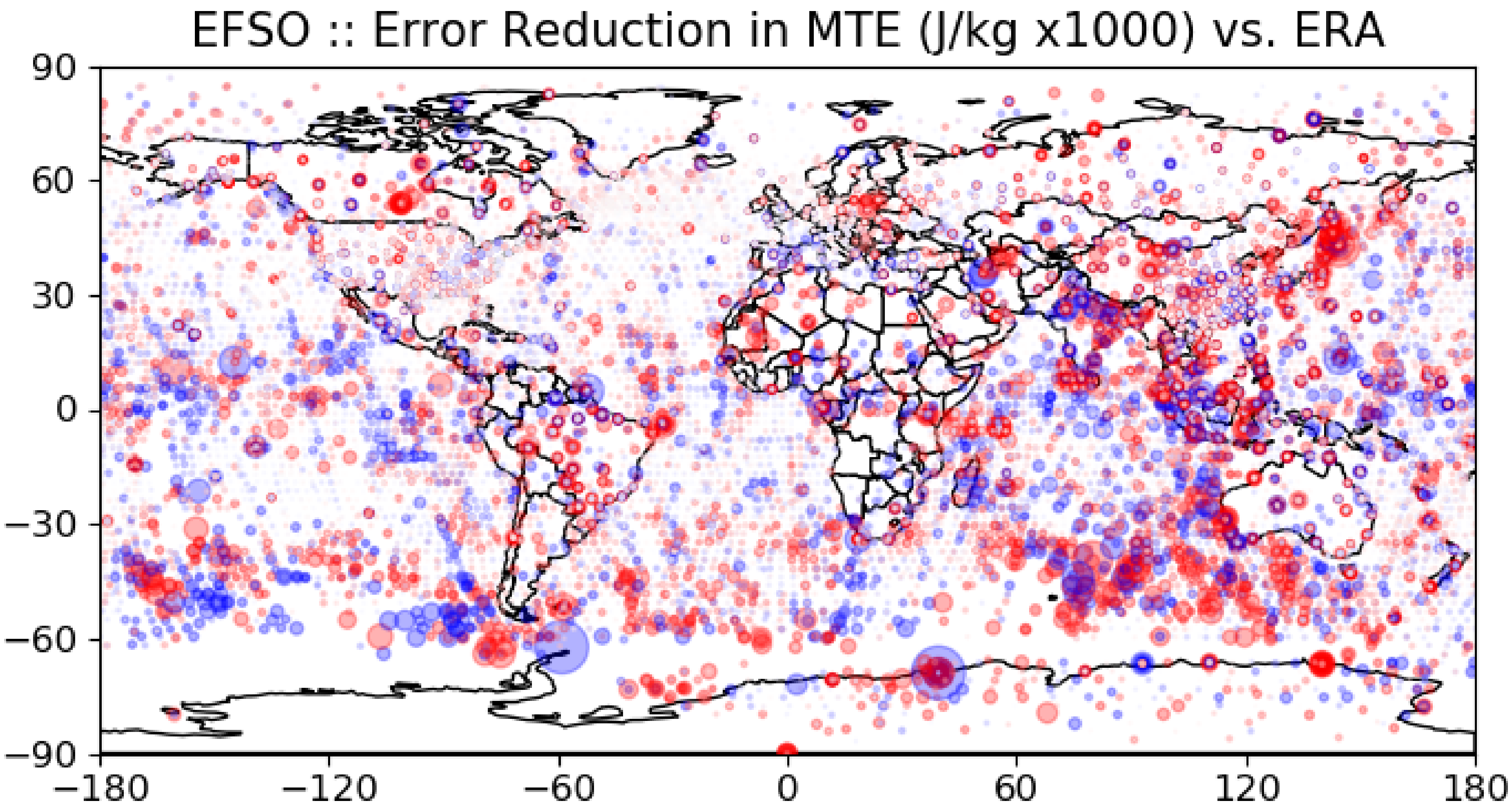
$$\approx -\frac{1}{p}\frac{1}{m-1}\delta\mathbf{y}^T_0\mathbf{R}^{-1}\mathbf{Y}^a_0\mathbf{Y}^{bT}_{t|0}\mathbf{R}^{-1}_t\left(\mathbf{d}_{t|0} + \mathbf{d}_{t|-6}\right)$$

obs-minus-FG   AN ptb in obs space   FCST ptb

EFSO with NICAM-LETKF



Estimated Impacts by EFSO



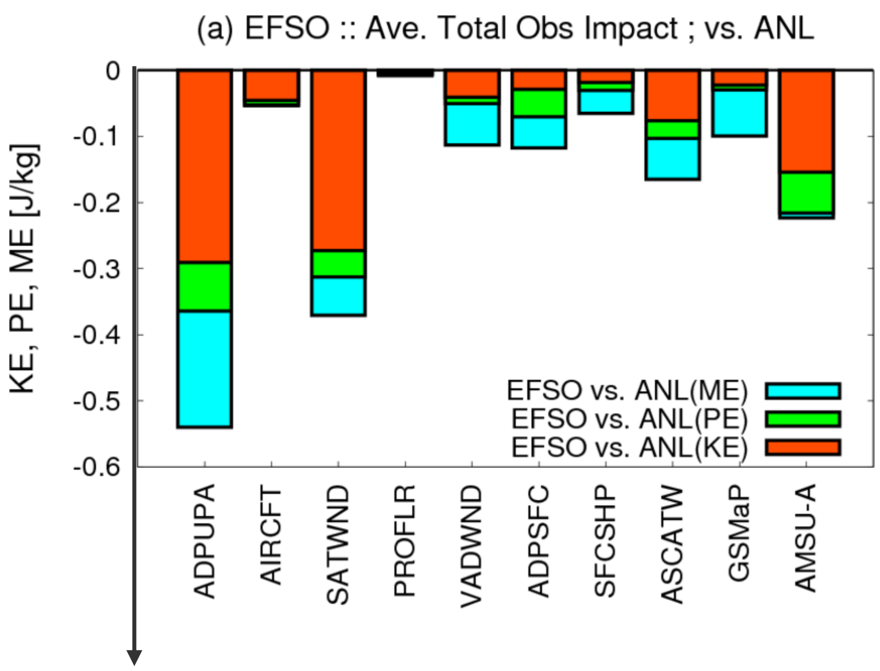
● ***Beneficial observation***

● ***Detrimental observation***

2014/07/11/00UTC; vs. ERA interim

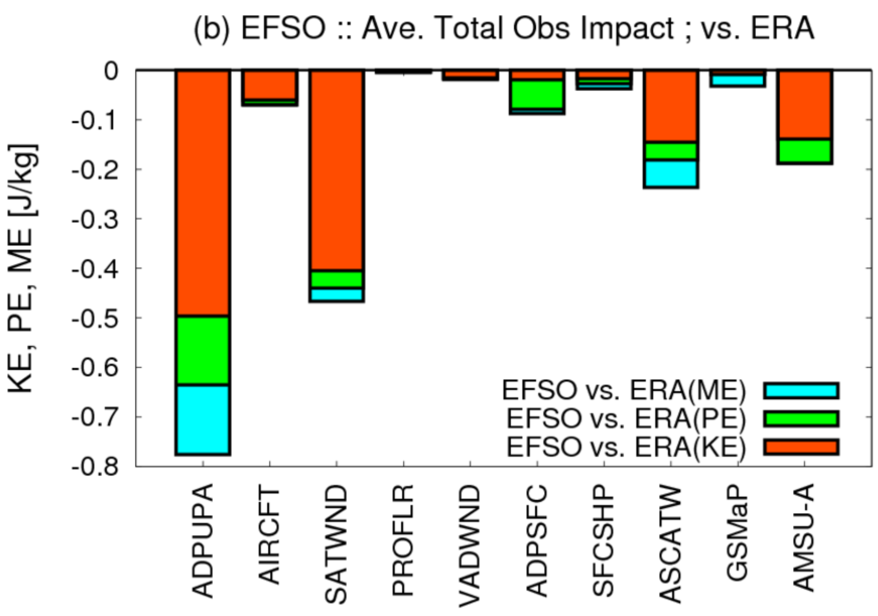
FCST MTE Error Reduction

vs. NICAM-LETKF



Positive Impact
(improving 6-hr FCST)

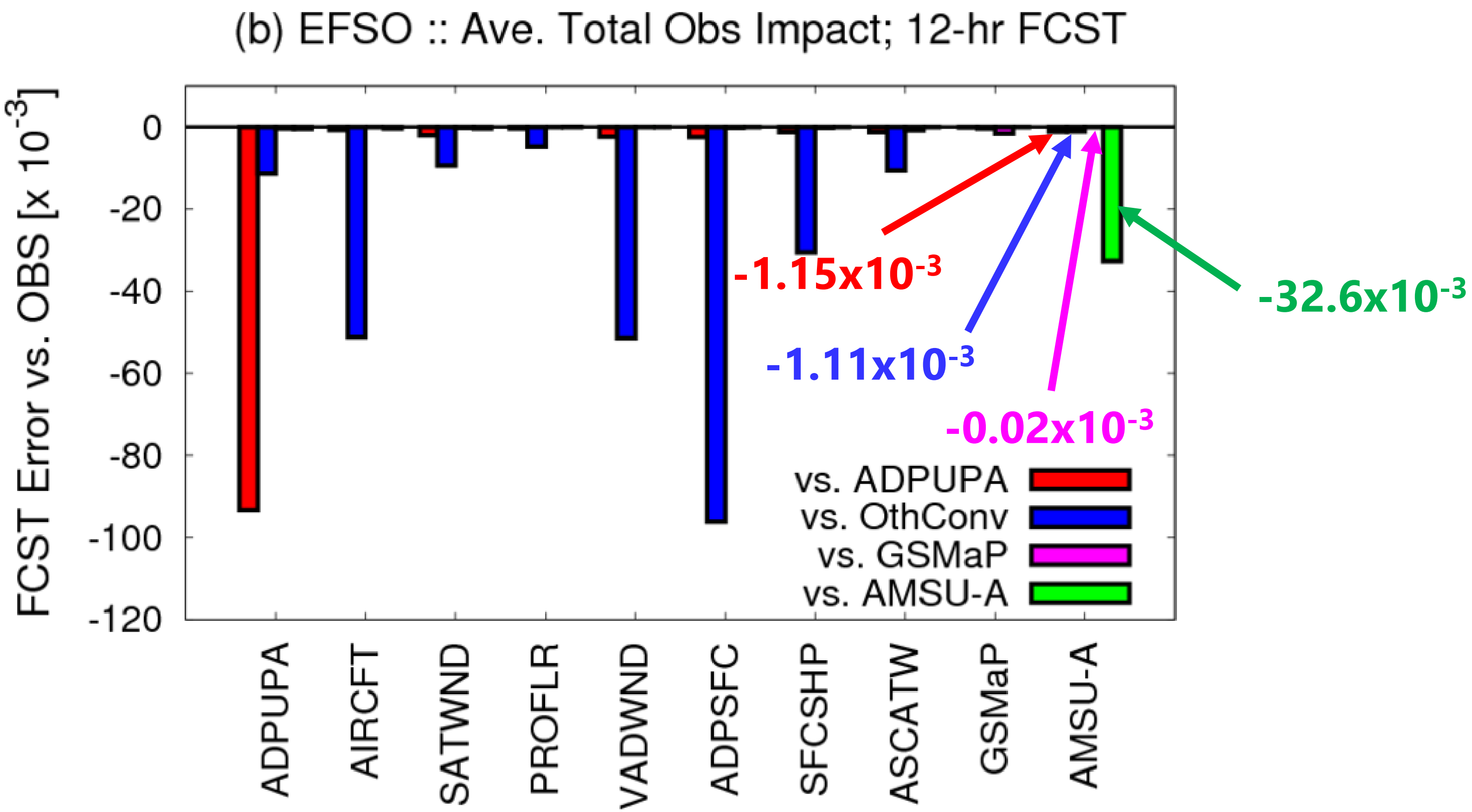
vs. ERA Interim



- **ME : Moist Energy**
- **PE : Potential Energy**
- **KE : Kinetic Energy**

Monthly average in July 2014; FT 06hr

FCST **NOD** Error Reduction



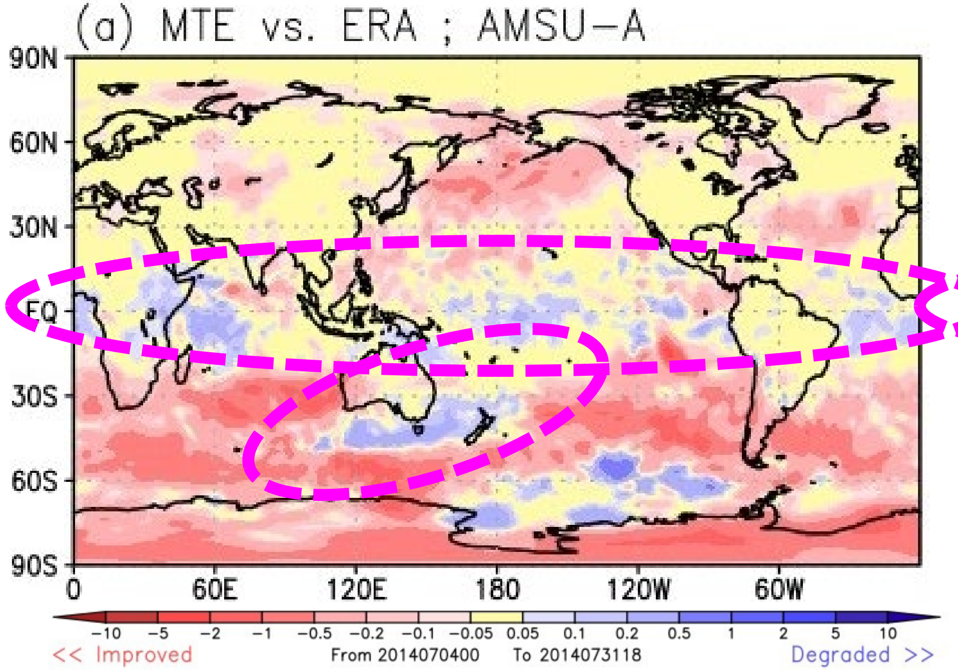
Each type of observations mainly contributes to the improvement of the observed variable.

Monthly average in July 2014; FT 12hr

Detection of detrimental AMSU-A radiances

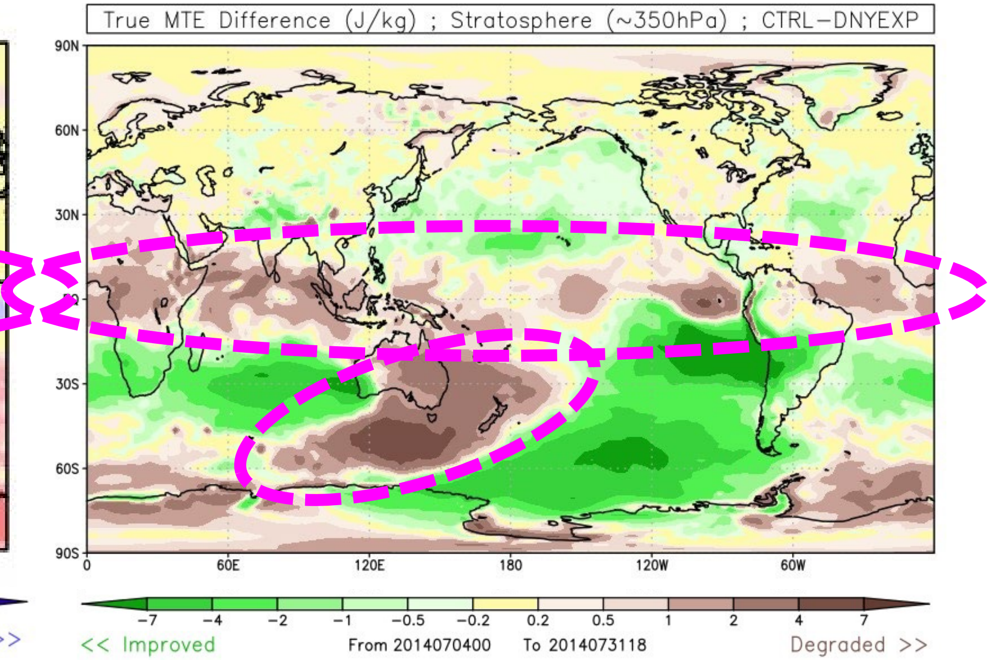
Impacts of AMSU-A on upper atmos. (~350hPa, vs. ERA)

EFSO Estimates



detrimental ➔

w/wo AMSU-A



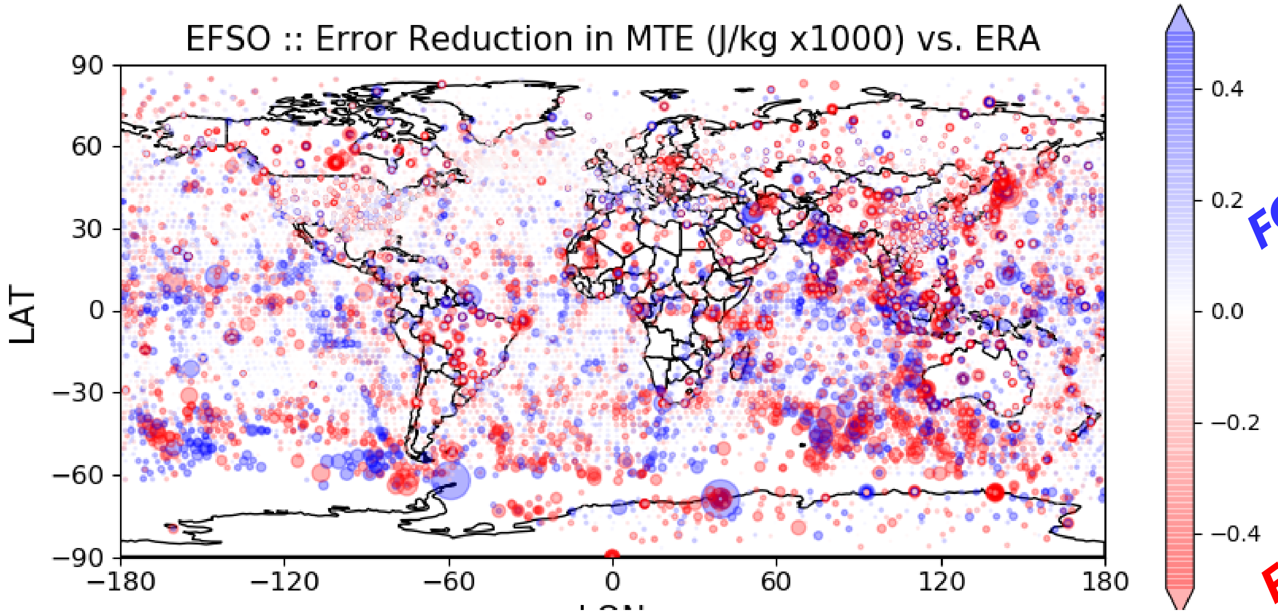
detrimental ➔

We can evaluate impacts of AMSU-A by EFSO without expensive w/wo OSE experiments

On Beneficial & Detrimental Obs

FCST Error Reduction (2014071100UTC)

vs. ERA

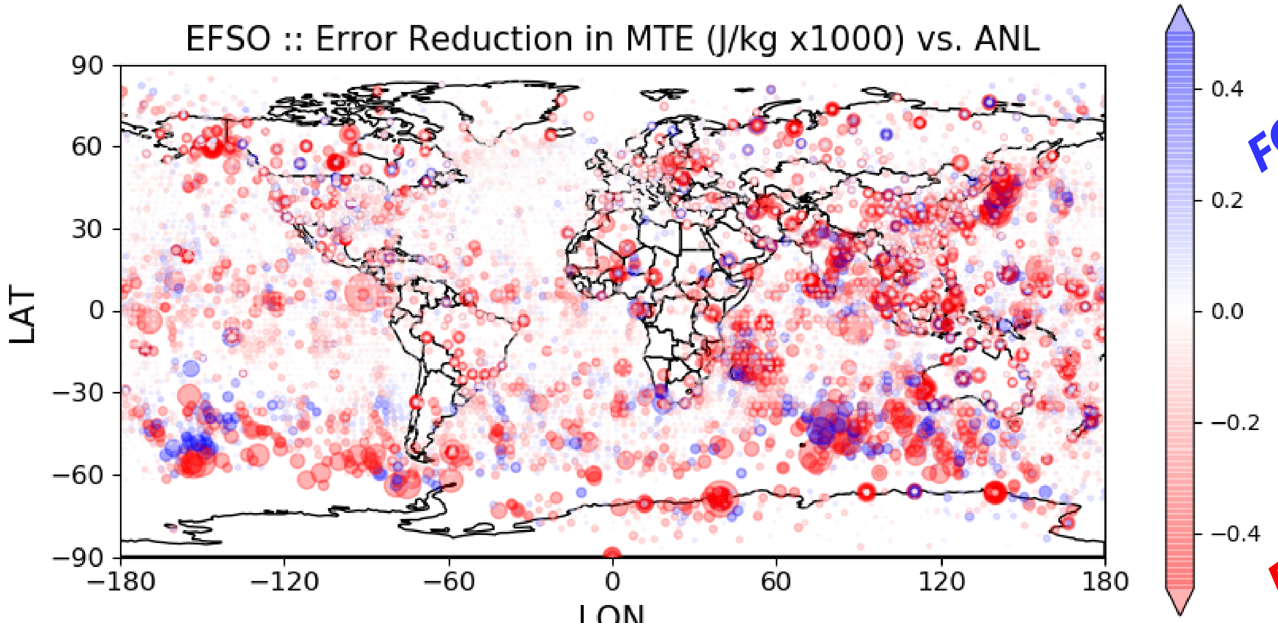


FCST degraded

FT 06hr

FCST improved

vs. NICAM -LETKF



FCST degraded

FT 06hr

FCST improved

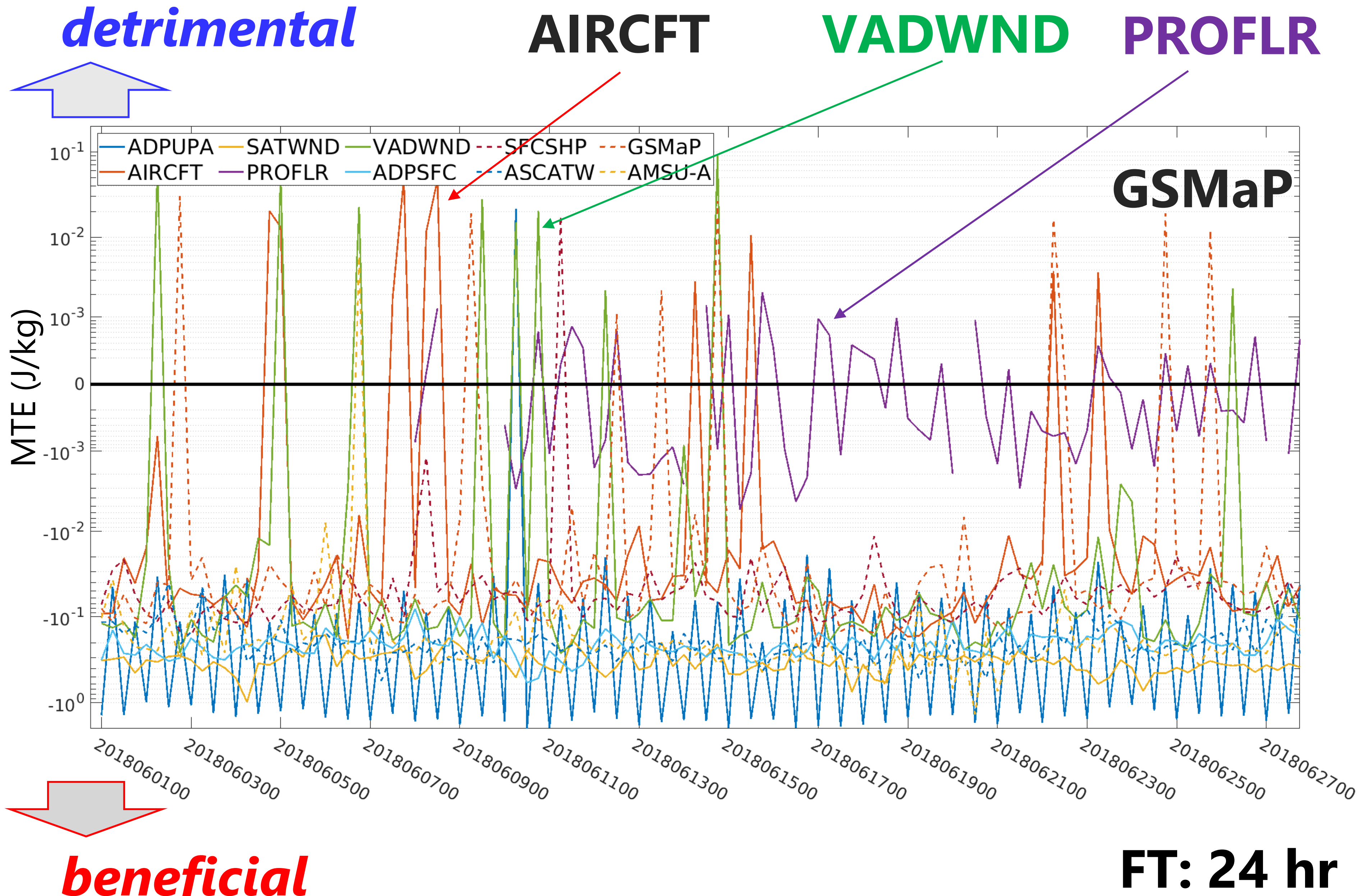
Fraction of Beneficial Obs

Ref.	Subseq. AN	ERA Interim	AMSU-A
FT 06hr	58.8 %	55.4 %	53.1 %
FT 12hr	56.1 %	54.2 %	53.2 %

FSO may overestimate observational impact with subsequent analyses for verification reference.

➔ It is good to use an independent anl for reference

Impacts in Time (vs. ERA Interim)



FT: 24 hr

Impacts in Space (vs. ERA Interim)

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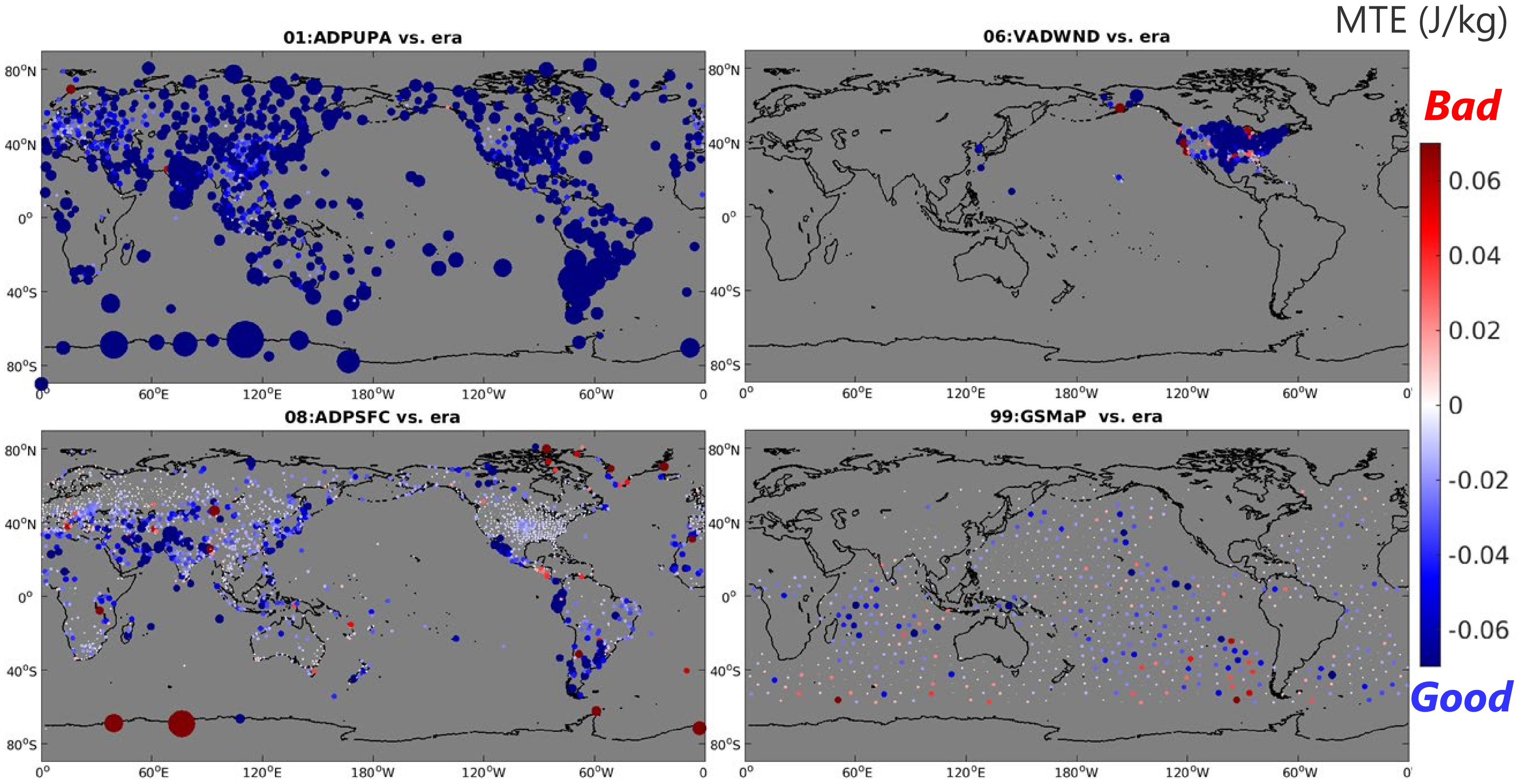

統計的に予報を改悪する観測を除くと良くなるのか？（課題）

FT: 24hr
SAMPLE: 2018060100-2018063018

A Case of
Heavy Rain in July 2018

Kotsuki, S.*, Terasaki, K., Kanemaru, K., Satoh, M., Kubota, T. and Miyoshi, T. (2019):
Predictability of Record-Breaking Rainfall in Japan in July 2018:
SOLA, 15A, 1-7. doi: 10.2151/sola.15A-001

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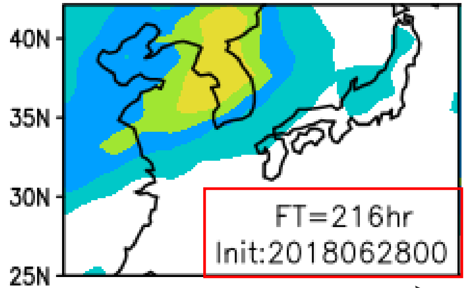
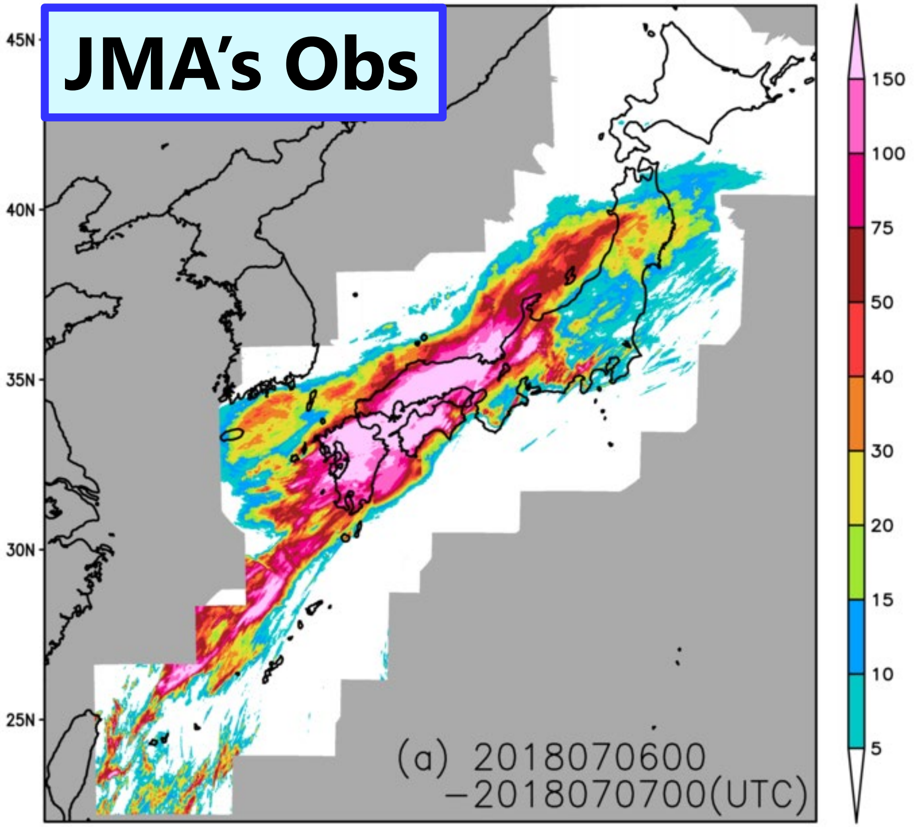
NICAM-LETKF Ens. FCSTs

NEXRA Precip. FCSTs (100−Ensemble) Period:201870600−201870700(UTC)

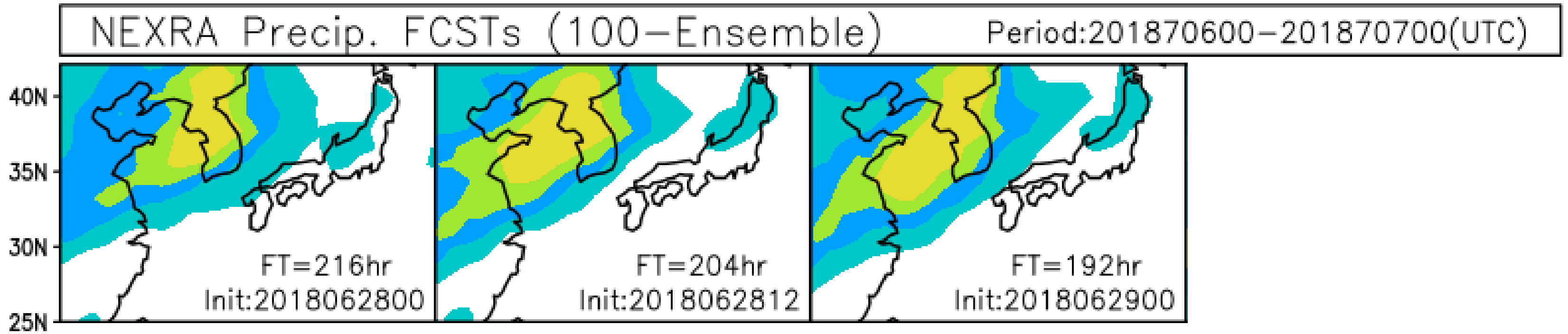


Forecast Lead Time (F
& Initialized Date

24-hr Accumulated Precip

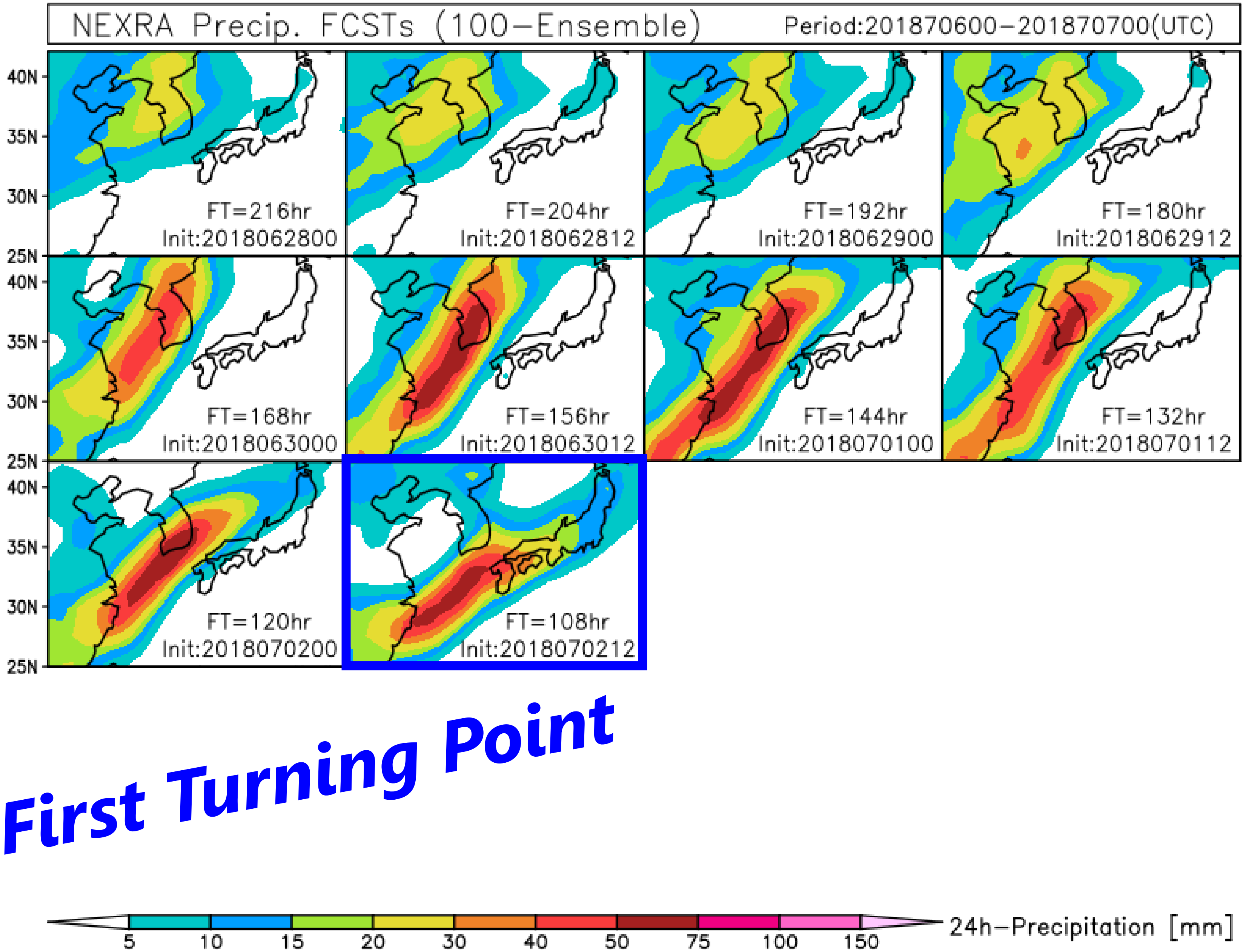


NICAM-LETKF Ens. FCSTs



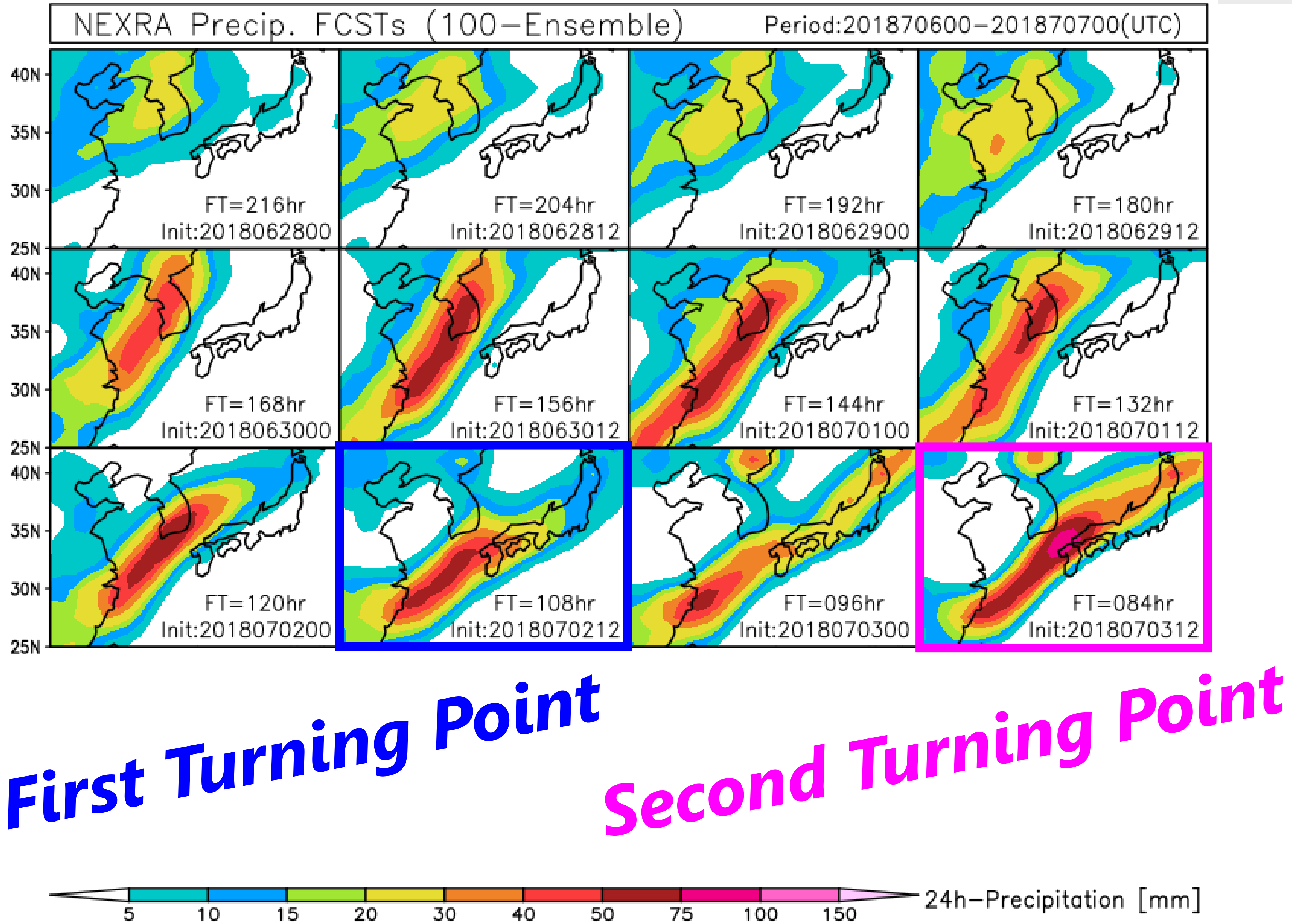
shifting initialized time

NICAM-LETKF Ens. FCSTs

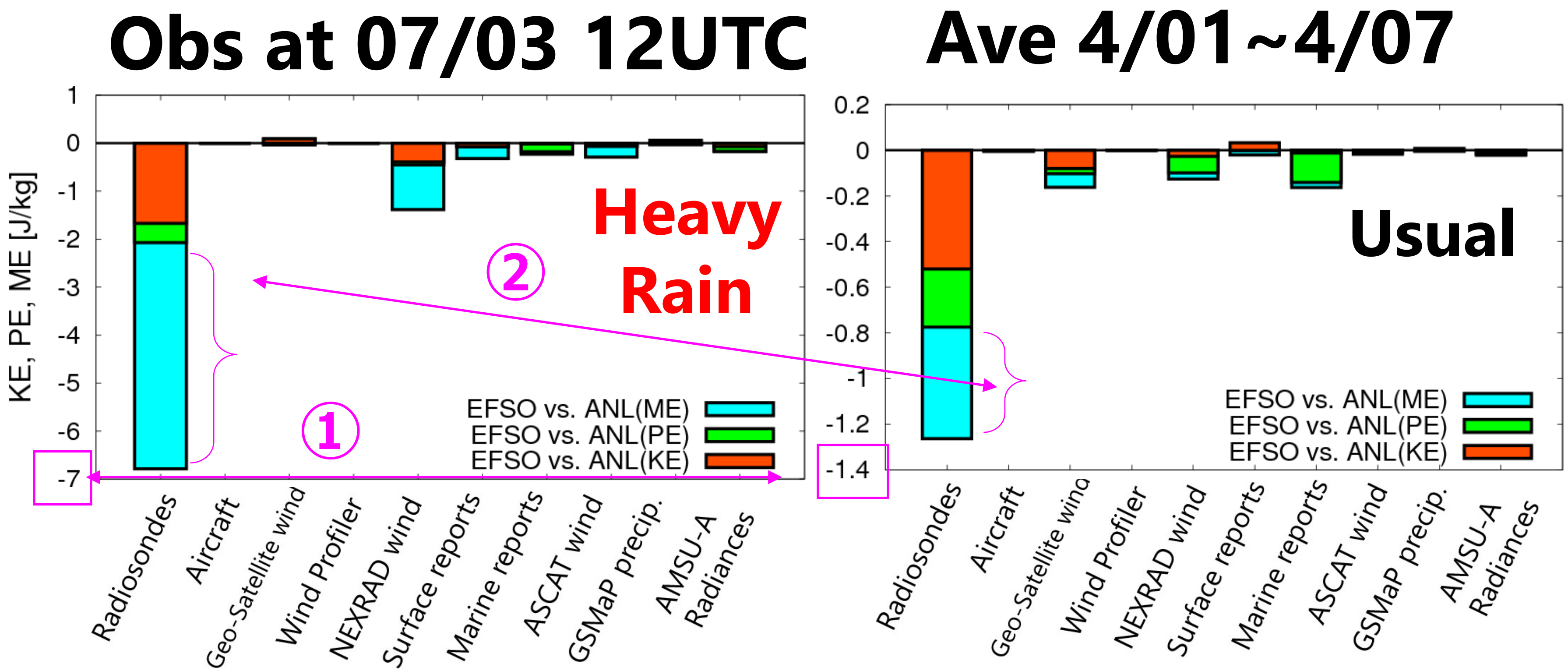


First Turning Point

NICAM-LETKF Ens. FCSTs



Obs Impacts



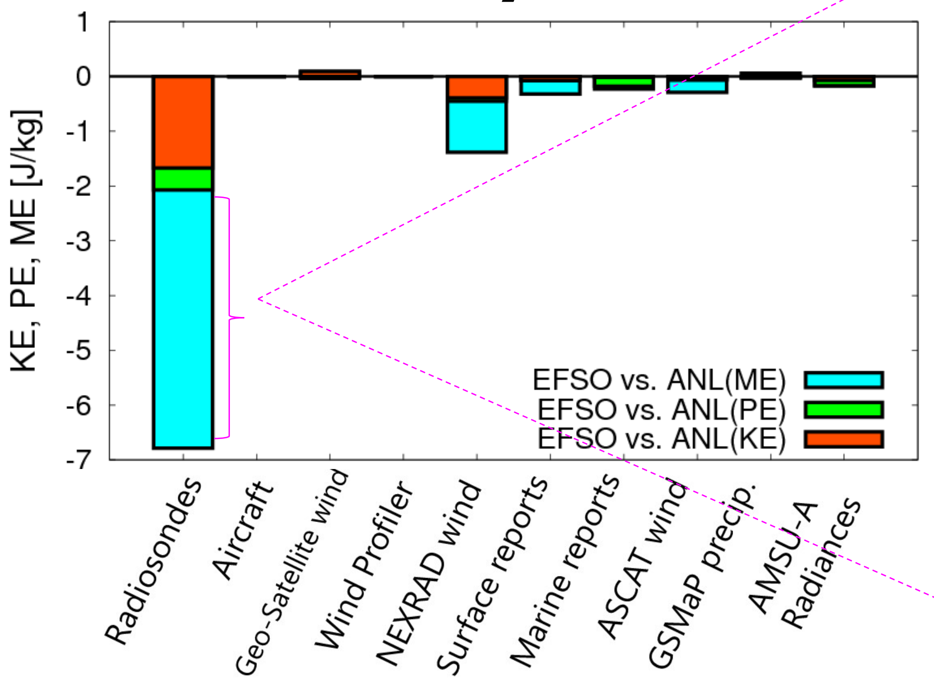
① Large impacts than usual ➔ rapid error growth
② Impacts on moist field is bigger

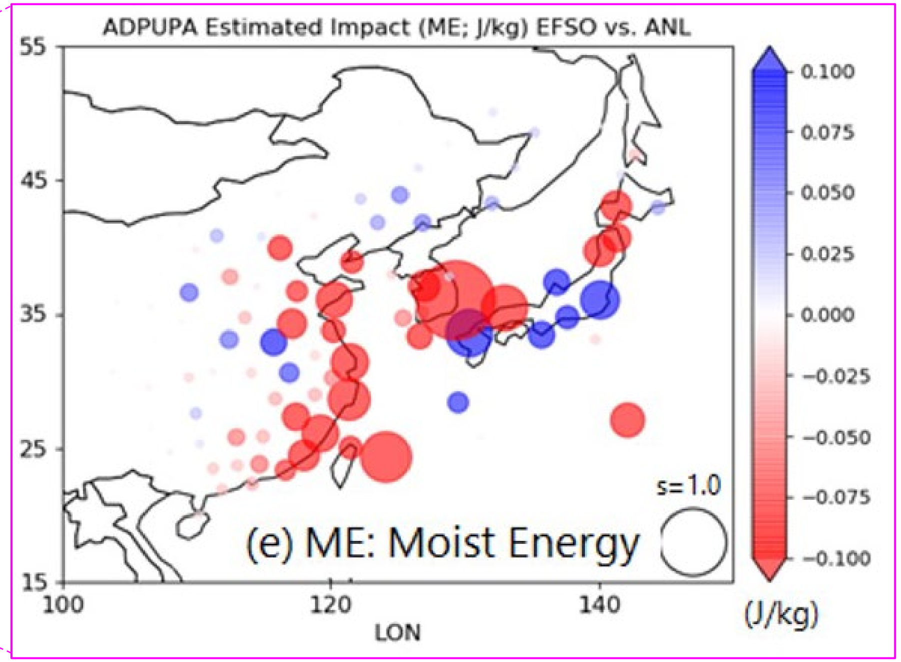
■ ME : Moist Energy (水蒸気の改善)
■ PE : Potential Energy (気温・気圧の改善)
■ KE : Kinetic Energy (風向風速の改善)

Radiosondes' Impacts

● : beneficial radiosondes
● : detrimental radiosondes

Obs at 07/03 12UTC





■ ME : Moist Energy (水蒸気の改善)
■ PE : Potential Energy (気温・気圧の改善)
■ KE : Kinetic Energy (風向風速の改善)

Thank you for your attention!

Presented by Shunji Kotsuki
(shunji.kotsuki@chiba-u.jp)

Further information is available at
https://kotsuki-lab.com/

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